



NIOS PYQ's SOLUTIONS

PREVIOUS YEARS' QUESTIONS & ANSWERS



APRIL 2026

Your Path to Success

(SET – A)

SECTION – A

Q1. Slope of a line which cuts off intercepts of equal length on the positive sides of two axes is:

- (A) -1 (B) 1
(C) 0 (D) 2

Answer: (A) -1

Q2. The coordinates of the foot of the perpendicular drawn from the point $(2, 3)$ on the line $y = 3x + 4$ is:

- (A) $\left(\frac{37}{10}, \frac{-1}{10}\right)$ (B) $\left(\frac{-1}{10}, \frac{37}{10}\right)$
(C) $\left(\frac{10}{37}, -10\right)$ (D) $\left(\frac{2}{3}, \frac{-1}{3}\right)$

Answer: (B) $\left(\frac{-1}{10}, \frac{37}{10}\right)$

Q3. If a circle passes through the points $(0, 0)$, $(a, 0)$ and $(0, b)$, then the centre of the circle is:

- (A) (a, b) (B) $\left(\frac{-a}{2}, \frac{-b}{2}\right)$
(C) $(-a, -b)$ (D) $\left(\frac{a}{2}, \frac{b}{2}\right)$

Answer: (D) $\left(\frac{a}{2}, \frac{b}{2}\right)$

Q4. The length of latus rectum of an ellipse $3x^2 + y^2 = 12$ is:

- (A) 8 (B) 4
(C) $\frac{2}{\sqrt{3}}$ (D) $\frac{4}{\sqrt{3}}$

Answer: (D) $\frac{4}{\sqrt{3}}$

Q5. The equation of a parabola whose vertex is $(-3, 0)$ and directrix is $x + 5 = 0$, is:

- (A) $x^2 = 8(y + 3)$ (B) $y^2 = 8(x + 3)$
(C) $x^2 = -8(y + 3)$ (D) $y^2 = -8(x + 3)$

Answer: (B) $y^2 = 8(x + 3)$

Q6. If $\begin{bmatrix} 4 & 3 \\ x & 5 \end{bmatrix} = \begin{bmatrix} y & z \\ 1 & 5 \end{bmatrix}$ then:

- (A) $x = 1, y = 3, z = 4$ (B) $x = 1, y = 4, z = 3$
(C) $x = 3, y = 1, z = 4$ (D) $x = 3, y = 4, z = 1$

Answer: (B) $x = 1, y = 4, z = 3$



Q7. In the matrix $A = (a_{ij}) = \begin{bmatrix} 0 & 6 & 3 \\ -6 & -3 & 4 \\ 2 & -2 & 0 \end{bmatrix}$ the value of $a_{13} a_{23} + a_{31}$ is:

- (A) 0 (B) 12
(C) -4 (D) 14

Answer: (D) 14

Q8. The cofactor of a_{23} in the determinant $\begin{vmatrix} 1 & 0 & 4 \\ 3 & 5 & -1 \\ 0 & 1 & 2 \end{vmatrix}$ is:

- (A) 1 (B) -1
(C) 2 (D) 4

Answer: (B) -1

Q9. If $f: \mathbb{R} \rightarrow \mathbb{R}$ is defined as $f(x) = x^4$ then:

- (A) f is one-one and onto (B) f is many-one and onto
(C) f is one-one but not onto (D) f is neither one-one nor onto

Answer: (D) f is neither one-one nor onto

Q10. If $f(x) = x^2 + 2$ and $g(x) = 1 - \frac{1}{1-x}$ ($x \neq 1$), then $f \circ g(x)$ is:

- (A) xx (B) $x^4 + 4x^2 + 6$
(C) $\frac{x^2}{(1-x)^2} + 2$ (D) $\frac{x^2+2}{x^2+1}$

Answer: (C) $\frac{x^2}{(1-x)^2} + 2$

Q11. The relation R in the set $A = \{1, 2, 3\}$ given by $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3)\}$ is:

- (A) reflexive only (B) symmetric only
(C) transitive only (D) symmetric and transitive

Answer: (A) reflexive only

Q12. If a binary operation is defined on the set $\mathbb{R}^+$ as $a * b = \frac{ab}{3}$ for all $a, b \in \mathbb{R}^+$, then the value of $(2 * 3) * 5$ is:

- (A) 7 (B) 10
(C) $\frac{13}{3}$ (D) $\frac{10}{3}$

Answer: (D) $\frac{10}{3}$

Q13. If $y = \log(2x + 3)$, then $\frac{dy}{dx}$ is:

- (A) $\frac{1}{2x+3}$ (B) $(2x + 3)^2$
(C) $\frac{2}{2x+3}$ (D) $\frac{1}{2(2x+3)}$



Answer: (C) $\frac{2}{2x+3}$

Q14. The derivative of $y = \sin^3 x$ w.r.t. x is:

- (A) $\cos^3 x$ (B) $3\sin^2 x$
(C) $\sin^2 x \cos x$ (D) $3\sin^2 x \cos x$

Answer: (D) $3\sin^2 x \cos x$

Q15. Which of the following is a vector quantity?

- (A) Distance (B) Velocity
(C) Time period (D) Work done

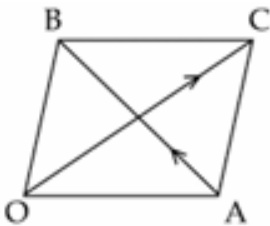
Answer: (B) Velocity

Q16. A unit vector in the direction of the vector $\vec{a} = 2\hat{i} + 3\hat{j} + \hat{k}$ is:

- (A) $\hat{i} + \hat{j} + \hat{k}$ (B) $5(2\hat{i} + 3\hat{j} + \hat{k})$
(C) $\frac{1}{\sqrt{6}}(2\hat{i} + 3\hat{j} + \hat{k})$ (D) $\frac{1}{\sqrt{14}}(2\hat{i} + 3\hat{j} + \hat{k})$

Answer: (D) $\frac{1}{\sqrt{14}}(2\hat{i} + 3\hat{j} + \hat{k})$

Q17. If OACB is a parallelogram with $\vec{OC} = \vec{a}$ and $\vec{AB} = \vec{b}$, then $\vec{OA}$ is equal to:



- (A) $\vec{a} + \vec{b}$ (B) $\vec{a} - \vec{b}$
(C) $\frac{1}{2}(\vec{a} - \vec{b})$ (D) $\frac{1}{2}(\vec{b} - \vec{a})$

Answer: (C) $\frac{1}{2}(\vec{a} - \vec{b})$

Q18. The degree of the differential equation $\frac{d^3 y}{dx^3} + 3\left(\frac{dy}{dx}\right)^2 = y^2 \log\left(\frac{d^2 y}{dx^2}\right)$ is:

- (A) 2 (B) 3
(C) 5 (D) not defined

Answer: (D) not defined

Q19. $\int_0^1 \frac{x}{x^2+1} dx$ is equal to:

- (A) $\frac{1}{2}$ (B) $\log 2$
(C) $2\log 2$ (D) $\frac{1}{2}\log 2$

Answer: (D) $\frac{1}{2}\log 2$



Q20. Converse of the statement "if a number is divisible by 9, then it is divisible by 3" is:

- (A) if a number is divisible by 3, then it is divisible by 9
- (B) if a number is divisible by 9, then it is not divisible by 3
- (C) if a number is not divisible by 3, it is not divisible by 9
- (D) if a number is not divisible by 9, then it is also not divisible by 3

Answer: (A) if a number is divisible by 3, then it is divisible by 9

Q21. Match Column-I statement with the correct option of Column-II:

A 2×3 matrix $[a_{ij}]$, whose elements are given by $a_{ij} = \frac{i-j}{i+j}$

Column – I	Column – II
(a) a_{23} is	(P) $\frac{1}{5}$
(b) a_{12} is	(Q) $\frac{-1}{5}$
	(R) $\frac{1}{3}$
	(S) $\frac{-1}{3}$

Answer: (a) $\rightarrow$ (Q); (b) $\rightarrow$ (S)

Q22. Fill in the blanks:

(a) The distance between the planes $\vec{r} \cdot (\hat{i} + \hat{j} - \hat{k}) + 4 = 0$ and $\vec{r} \cdot (\hat{i} + \hat{j} - \hat{k}) + 5 = 0$ is _____ units. $\frac{1}{\sqrt{3}}$

(b) The order of the differential equation $y = \frac{dy}{dx} + \left(\frac{dy}{dx}\right)^{-1}$ is _____. **Order = 1**

Q23. Write "True" for correct statement and "False" for incorrect statement:

(a) The function $f: \mathbb{R} \rightarrow \mathbb{R}$ defined as $f(x) = \begin{cases} 1, & \text{if } x \neq 0 \\ 2, & \text{if } x = 0 \end{cases}$ is continuous at $x = 0$ **False**

(b) The function $f: \mathbb{R} \rightarrow \mathbb{R}$ defined as $f(x) = |x|$ is continuous at all points of $\mathbb{R}$. **True**

Q24. Write the negation of each of the following statements:

(a) **The capital of Gujarat is Ahmedabad.**

Answer: The negation of a statement p is "not p " ($\sim p$).

Negation: The capital of Gujarat is not Ahmedabad.

(b) **All circles are congruent.**

Answer: The negation of a universal statement "All A are B" is "It is not the case that all A are B," which can be stated as "There exists at least one A that is not B."



Negation: It is false that all circles are congruent. (Equivalently: Not all circles are congruent / There exists at least one pair of circles that are not congruent.)

Q25. Fill in the blanks:

- (a) The function f defined as $f(x) = x - \cos x$ is _____ function for all x . **Increasing**
- (b) The local minima of the function $f(x) = 2x^3 - 3x^2 - 12x + 8$ exists at _____. **$x = 2$**
- (c) The slope of the normal to the curve $x^3 + x^2 + 3xy + y^2 - 5 = 0$ at $(1, 1)$ is _____. **$-\frac{5}{8}$**
- (d) The tangent to a curve $y = f(x)$ at the point (x_1, y_1) is parallel to x -axis, if _____. **$\left(\frac{dy}{dx}\right)_{(x_1, y_1)} = 0$**

Q26. Fill in the blanks:

- (a) If $y = e^{\sin^{-1} x}$, then $\frac{dy}{dx} =$ _____. **$\frac{e^{\sin^{-1} x}}{\sqrt{1-x^2}}$**
- (b) If $x = at^2, y = 2at$, then $\frac{dy}{dx} =$ _____. **$\frac{1}{t}$**
- (c) The number of arbitrary constants in the particular solution of a differential equation of third order is _____. **0 (zero)**
- (d) $\int e^x \left(\frac{1}{x} - \frac{1}{x^2} \right) dx =$ _____. **$\frac{e^x}{x} + C$**

Q27. Write "True" for correct statement and "False" for incorrect statement:

- (a) Two matrices are comparable if each one of them contains as many rows and columns as the other. **True**
- (b) Matrix A is said to singular if $|A| \neq 0$. **False**
- (c) The adjoint of the matrix $A = \begin{bmatrix} 2 & 3 \\ 1 & -2 \end{bmatrix}$ is $\begin{bmatrix} -2 & -3 \\ -1 & 2 \end{bmatrix}$. **True**
- (d) The inverse of a skew symmetric matrix of order 3 is a diagonal matrix. **False**

Q28. Answer the following questions:

- (a) Check if the relation R in the set $A = \{1, 2, 3\}$ given by $R = \{(1, 2), (2, 1)\}$ is an equivalence relation.

Answer: For R to be an equivalence relation, it must be reflexive, symmetric, and transitive.

Reflexive: We need $(1,1), (2,2), (3,3) \in R$. None of these are present in R . So R is not reflexive. Since R is not reflexive, R is not an equivalence relation.

(For completeness: Symmetric $(1,2) \in R$ and $(2,1) \in R$, so R is symmetric. Transitive $(1,2) \in R, (2,1) \in R \Rightarrow (1,1)$ should be in R for transitivity but it's not, so R is not transitive either.) R is NOT an equivalence relation (since it is not reflexive).



(b) Check if the function $f: \mathbb{R} \rightarrow \mathbb{R}$ defined as $f(x) = 3 - 4x$ is one-one and onto.

Answer: One-one: Let $f(x_1) = f(x_2)$

$$3 - 4x_1 = 3 - 4x_2 \Rightarrow -4x_1 = -4x_2 \Rightarrow x_1 = x_2$$

So f is one-one.

Onto: Let $y \in \mathbb{R}$ (co-domain) be arbitrary. We need to find $x \in \mathbb{R}$ such that $f(x) = y$.

$$y = 3 - 4x \Rightarrow x = \frac{3-y}{4}$$

Since $x = \frac{3-y}{4} \in \mathbb{R}$ for every $y \in \mathbb{R}$, f is onto.

$f(x) = 3 - 4x$ is both one-one and onto.

(c) Find: $\int \sin^{-1}(\cos x) dx$

Answer: We know that $\cos x = \sin\left(\frac{\pi}{2} - x\right)$

$$\sin^{-1}(\cos x) = \sin^{-1}\left[\sin\left(\frac{\pi}{2} - x\right)\right] = \frac{\pi}{2} - x$$

(valid for $x \in [0, \pi]$ the principal range consideration)

$$\int \sin^{-1}(\cos x) dx = \int \left(\frac{\pi}{2} - x\right) dx = \frac{\pi}{2}x - \frac{x^2}{2} + C$$

$$\frac{\pi x}{2} - \frac{x^2}{2} + C$$

(d) Using differentials, find the approximate value of $\sqrt{26}$ up to one place of decimal.

Answer: Let $f(x) = \sqrt{x}$. Take $x = 25$ (nearest perfect square) and $\Delta x = 1$.

$$f'(x) = \frac{1}{2\sqrt{x}}$$

$$f(x + \Delta x) \approx f(x) + f'(x) \cdot \Delta x$$

$$\sqrt{26} \approx \sqrt{25} + \frac{1}{2\sqrt{25}} \times 1$$

$$\sqrt{26} \approx 5 + \frac{1}{2 \times 5} = 5 + \frac{1}{10} = 5 + 0.1 = 5.1$$

$$\sqrt{26} \approx 5.1$$

Q29. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$ be two vectors. Then:

(a) Find $\vec{a} \times \vec{b}$

$$\text{Answer: } \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{vmatrix}$$

$$= \hat{i}[(1)(1) - (1)(-1)] - \hat{j}[(1)(1) - (1)(1)] + \hat{k}[(1)(-1) - (1)(1)]$$

$$= \hat{i}[1 + 1] - \hat{j}[1 - 1] + \hat{k}[-1 - 1]$$

$$= 2\hat{i} - 0\hat{j} - 2\hat{k}$$

$$\vec{a} \times \vec{b} = 2\hat{i} - 2\hat{k}$$



(b) Find the angle between $\vec{a}$ and $\vec{b}$

Answer: $\vec{a} \cdot \vec{b} = (1)(1) + (1)(-1) + (1)(1) = 1 - 1 + 1 = 1$

$$|\vec{a}| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}, |\vec{b}| = \sqrt{1^2 + (-1)^2 + 1^2} = \sqrt{3}$$

$$\cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} = \frac{1}{\sqrt{3} \times \sqrt{3}} = \frac{1}{3}$$

$$\theta = \cos^{-1} \left(\frac{1}{3} \right)$$

(c) Find the projection of $\vec{a}$ on the vector $\vec{b}$

Answer: Projection of $\vec{a}$ on $\vec{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} = \frac{1}{\sqrt{3}}$

(d) Check, if $\vec{a} \times \vec{b} = \vec{b} \times \vec{a}$

Answer: We know the vector product is anti-commutative: $\vec{b} \times \vec{a} = -(\vec{a} \times \vec{b})$. We already found $\vec{a} \times \vec{b} = 2\hat{i} - 2\hat{k} \neq \vec{0}$ (unless the vectors are parallel, which they aren't here).

So $\vec{b} \times \vec{a} = -2\hat{i} + 2\hat{k} \neq \vec{a} \times \vec{b}$ (since $2\hat{i} - 2\hat{k} \neq -2\hat{i} + 2\hat{k}$).

$\vec{a} \times \vec{b} \neq \vec{b} \times \vec{a}$ (they are negatives of each other, not equal, since $\vec{a} \times \vec{b} \neq \vec{0}$)

(e) Find the area of the parallelogram having $\vec{a}$ and $\vec{b}$ as its diagonals

Answer: Area of parallelogram with diagonals $\vec{a}, \vec{b}$ is: Area = $\frac{1}{2} |\vec{a} \times \vec{b}|$

$$|\vec{a} \times \vec{b}| = |2\hat{i} - 2\hat{k}| = \sqrt{2^2 + 0^2 + (-2)^2}$$

$$= \sqrt{4 + 4} = \sqrt{8} = 2\sqrt{2}$$

$$\text{Area} = \frac{1}{2} \times 2\sqrt{2} = \sqrt{2}$$

(f) Check if $(\vec{a} \times \vec{b})^2 = (\vec{a})^2(\vec{b})^2 - (\vec{a} \cdot \vec{b})^2$

Answer: This is Lagrange's identity, which should always hold.

$$\text{LHS: } (\vec{a} \times \vec{b})^2 = |\vec{a} \times \vec{b}|^2 = |2\hat{i} - 2\hat{k}|^2 = 4 + 4 = 8$$

$$\text{RHS: } |\vec{a}|^2 |\vec{b}|^2 - (\vec{a} \cdot \vec{b})^2 = (\sqrt{3})^2 (\sqrt{3})^2 - (1)^2$$

$$= 3 \times 3 - 1 = 9 - 1 = 8$$

Since LHS = RHS = 8

Verified the identity holds true.

SECTION - B

Q30. Using matrix method, solve the following system of linear equations:

$$5x - 7y = 3$$

$$x + 3y = 5$$



Answer: Write in matrix form $AX = B$: $A = \begin{bmatrix} 5 & -7 \\ 1 & 3 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$, $B = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$

$$|A| = 5(3) - (-7)(1) = 15 + 7 = 22 \neq 0$$

Since $|A| \neq 0$, A^{-1} exists.

$$\text{adj}(A) = \begin{bmatrix} 3 & 7 \\ -1 & 5 \end{bmatrix}$$

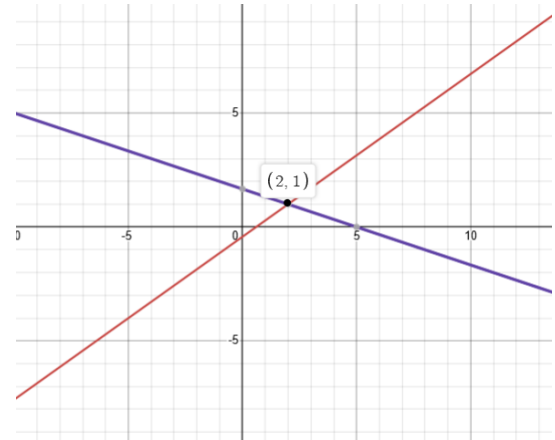
$$A^{-1} = \frac{1}{|A|} \text{adj}(A) = \frac{1}{22} \begin{bmatrix} 3 & 7 \\ -1 & 5 \end{bmatrix}$$

$$X = A^{-1}B = \frac{1}{22} \begin{bmatrix} 3 & 7 \\ -1 & 5 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

$$= \frac{1}{22} \begin{bmatrix} 9 + 35 \\ -3 + 25 \end{bmatrix}$$

$$= \frac{1}{22} \begin{bmatrix} 44 \\ 22 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$x = 2, y = 1$$



Q31. Simplify: (i) $\cos(\sin^{-1} x)$

Answer: Let $\sin^{-1} x = \theta \Rightarrow \sin \theta = x$

$$\cos \theta = \sqrt{1 - \sin^2 \theta} = \sqrt{1 - x^2}$$

$$\sqrt{1 - x^2}$$

OR

(ii) $\cot(\text{cosec}^{-1} x)$

Answer: Let $\text{cosec}^{-1} x = \theta \Rightarrow \text{cosec} \theta = x$

$$\text{We know: } \text{cosec}^2 \theta - \cot^2 \theta = 1 \Rightarrow \cot^2 \theta = \text{cosec}^2 \theta - 1 = x^2 - 1$$

$$\cot \theta = \sqrt{x^2 - 1}$$

$$\sqrt{x^2 - 1}$$

Q32. Find: (i) $\lim_{x \rightarrow 3} \left(\frac{x^4 - 81}{2x^2 - 5x - 3} \right)$

Answer: At $x = 3$: numerator = $81 - 81 = 0$;

denominator = $18 - 15 - 3 = 0$ (form $0/0$)

$$\text{Numerator: } x^4 - 81 = (x^2 - 9)(x^2 + 9) = (x - 3)(x + 3)(x^2 + 9)$$

$$\text{Denominator: } 2x^2 - 5x - 3 = 2x^2 - 6x + x - 3 = 2x(x - 3) + 1(x - 3) = (2x + 1)(x - 3)$$

$$\lim_{x \rightarrow 3} \frac{(x-3)(x+3)(x^2+9)}{(2x+1)(x-3)}$$

$$= \lim_{x \rightarrow 3} \frac{(x+3)(x^2+9)}{2x+1}$$

$$= \frac{(3+3)(9+9)}{2(3)+1} = \frac{6 \times 18}{7} = \frac{108}{7}$$



OR

(ii) $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2}$

Answer: Using $1 - \cos x = 2\sin^2\left(\frac{x}{2}\right)$:

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{2\sin^2(x/2)}{x^2} &= \lim_{x \rightarrow 0} \frac{2\sin^2(x/2)}{4(x/2)^2} \\ &= \frac{1}{2} \lim_{x \rightarrow 0} \left(\frac{\sin(x/2)}{x/2} \right)^2 = \frac{1}{2} \times (1)^2 = \frac{1}{2} \end{aligned}$$

Q33. Find the coordinates of the foci, the vertices, the eccentricity and the length of the latus rectum of the conic:

(i) $\frac{x^2}{16} - \frac{y^2}{9} = 1$

Answer: This is a hyperbola with $a^2 = 16, b^2 = 9$ so $a = 4, b = 3$.

Vertices: $(\pm a, 0) = (\pm 4, 0)$

$$c^2 = a^2 + b^2 = 16 + 9 = 25 \Rightarrow c = 5$$

Foci: $(\pm c, 0) = (\pm 5, 0)$

Eccentricity: $e = \frac{c}{a} = \frac{5}{4}$

Length of latus rectum: $\frac{2b^2}{a} = \frac{2 \times 9}{4} = \frac{18}{4} = \frac{9}{2}$

OR

(ii) $4x^2 + 9y^2 = 36$

Answer: Dividing by 36: $\frac{x^2}{9} + \frac{y^2}{4} = 1$

This is an ellipse with $a^2 = 9, b^2 = 4$ (since $9 > 4$, major axis along xx-axis), so $a = 3, b = 2$.

Vertices: $(\pm a, 0) = (\pm 3, 0)$

$$c^2 = a^2 - b^2 = 9 - 4 = 5 \Rightarrow c = \sqrt{5}$$

Foci: $(\pm \sqrt{5}, 0)$

Eccentricity: $e = \frac{c}{a} = \frac{\sqrt{5}}{3}$

Length of latus rectum: $\frac{2b^2}{a} = \frac{2 \times 4}{3} = \frac{8}{3}$

Q34. Without expanding the determinant, prove that
$$\begin{vmatrix} 0 & a & -b \\ -a & 0 & -c \\ b & c & 0 \end{vmatrix} = 0.$$

Answer: Let $\Delta = \begin{vmatrix} 0 & a & -b \\ -a & 0 & -c \\ b & c & 0 \end{vmatrix}$



This is a skew-symmetric matrix (since $A^T = -A$: transposing gives $\begin{bmatrix} 0 & -a & b \\ a & 0 & c \\ -b & -c & 0 \end{bmatrix}$ which is exactly $-A$).

We know that for any square matrix A , $|A^T| = |A|$

Also, if $A^T = -A$ (skew-symmetric), then $|A^T| = |-A| = (-1)^3 |A| = -|A|$ (since it's a 3×3 , i.e., odd order matrix).

So $|A| = |A^T| = -|A|$

$\Rightarrow |A| = -|A| \Rightarrow 2|A| = 0 \Rightarrow |A| = 0$

Hence proved: $\Delta = 0$

Q35. Using elementary row transformation, find the inverse of the matrix $A = \begin{bmatrix} 3 & 5 \\ 4 & 7 \end{bmatrix}$

Answer: Write $A = IA$: $\begin{bmatrix} 3 & 5 \\ 4 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

$$R_1 \rightarrow R_1 - R_2: \begin{bmatrix} -1 & -2 \\ 4 & 7 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} A$$

$$R_1 \rightarrow -R_1: \begin{bmatrix} 1 & 2 \\ 4 & 7 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 0 & 1 \end{bmatrix} A$$

$$R_2 \rightarrow R_2 - 4R_1: \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 4 & -3 \end{bmatrix} A$$

$$R_2 \rightarrow -R_2: \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ -4 & 3 \end{bmatrix} A$$

$$R_1 \rightarrow R_1 - 2R_2: \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 - 2(-4) & 1 - 2(3) \\ -4 & 3 \end{bmatrix} A = \begin{bmatrix} 7 & -5 \\ -4 & 3 \end{bmatrix} A$$

Q36. Find: $\int \frac{1}{x^2-9} dx$

Answer: Using the standard formula $\int \frac{dx}{x^2-a^2}$

$$= \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + C, \text{ with } a = 3:$$

$$\int \frac{1}{x^2-9} dx = \frac{1}{6} \log \left| \frac{x-3}{x+3} \right| + C$$

OR

Evaluate: $\int_0^2 3^x dx$

Answer: $\int_0^2 3^x dx = \left[\frac{3^x}{\log 3} \right]_0^2$

$$= \frac{3^2}{\log 3} - \frac{3^0}{\log 3}$$

$$= \frac{9-1}{\log 3}$$

$$= \frac{8}{\log 3}$$



Q37. Find the value of k for which the line $(k - 3)x - (4 - k^2)y + k^2 - 7k + 6 = 0$ is parallel to x -axis.

Answer: A line is parallel to the x -axis when the coefficient of x is zero (and coefficient of y is non-zero), i.e., the line has the form $y = \text{constant}$.

So we need: $k - 3 = 0 \Rightarrow k = 3$

Check: at $k = 3$, coefficient of $y = -(4 - 9) = -(-5) = 5 \neq 0$, so the line does reduce to a valid horizontal line. $k = 3$

Q38. Find the Cartesian equation of the line which passes through the point $(1, -1, 5)$ and is parallel to the line $\frac{x-2}{3} = \frac{y-5}{-2}, z = -1$.

Answer: The given line $\frac{x-2}{3} = \frac{y-5}{-2}, z = -1$ can be rewritten in standard symmetric form as: $\frac{x-2}{3} = \frac{y-5}{-2} = \frac{z-(-1)}{0}$

So its direction ratios are $3, -2, 0$.

Since the required line is parallel to this line, it has the same direction ratios: $3, -2, 0$.

The required line passes through $(1, -1, 5)$: $\frac{x-1}{3} = \frac{y-(-1)}{-2} = \frac{z-5}{0}$

Q39. Evaluate: $\int_{-5}^5 |x + 2| dx$

Answer: $|x + 2| = 0$ when $x = -2$.

For $x < -2$: $x + 2 < 0$, so $|x + 2| = -(x + 2)$

For $x \geq -2$: $x + 2 \geq 0$, so $|x + 2| = x + 2$

$$\int_{-5}^5 |x + 2| dx = \int_{-5}^{-2} -(x + 2) dx + \int_{-2}^5 (x + 2) dx$$

$$\text{First integral: } \int_{-5}^{-2} -(x + 2) dx = -\left[\frac{x^2}{2} + 2x\right]_{-5}^{-2}$$

$$= -\left[\left(\frac{4}{2} - 4\right) - \left(\frac{25}{2} - 10\right)\right]$$

$$= -[(2 - 4) - (12.5 - 10)]$$

$$= -[-2 - 2.5]$$

$$= -(-4.5) = 4.5$$

$$\text{Second integral: } \int_{-2}^5 (x + 2) dx = \left[\frac{x^2}{2} + 2x\right]_{-2}^5$$

$$= \left(\frac{25}{2} + 10\right) - \left(\frac{4}{2} - 4\right)$$

$$= (12.5 + 10) - (2 - 4)$$

$$= 22.5 - (-2) = 24.5$$

$$\text{Total: } 4.5 + 24.5 = 29$$



OR

Determine the area enclosed by the curves $y = x^2$ and $y = x + 2$, using integration.

Answer: Find intersection points: $x^2 = x + 2 \Rightarrow x^2 - x - 2 = 0 \Rightarrow (x - 2)(x + 1) = 0$

$$x = -1 \text{ or } x = 2$$

For $x \in (-1, 2)$ the line $y = x + 2$ lies above the parabola $y = x^2$ (check at $x = 0$: line gives 2, parabola gives 0).

$$\text{Area} = \int_{-1}^2 [(x + 2) - x^2] dx = \left[\frac{x^2}{2} + 2x - \frac{x^3}{3} \right]_{-1}^2$$

$$\text{At } x = 2: \frac{4}{2} + 4 - \frac{8}{3} = 2 + 4 - \frac{8}{3} = 6 - \frac{8}{3} = \frac{18-8}{3} = \frac{10}{3}$$

$$\text{At } x = -1: \frac{1}{2} - 2 - \frac{-1}{3} = \frac{1}{2} - 2 + \frac{1}{3} = \frac{3-12+2}{6} = \frac{-7}{6}$$

$$\text{Area} = \frac{10}{3} - \left(\frac{-7}{6} \right) = \frac{10}{3} + \frac{7}{6}$$

$$= \frac{20}{6} + \frac{7}{6} = \frac{27}{6} = \frac{9}{2}$$

$$\frac{9}{2} \text{ square units}$$

Q40. Find the Cartesian equation of the plane passing through the points $(1, 1, 0)$, $(1, 2, 1)$ and $(-2, 2, -1)$. Also, write the vector equation of the plane.

Answer: Let the plane pass through $A(1, 1, 0)$, $B(1, 2, 1)$, $C(-2, 2, -1)$.

$$\overrightarrow{AB} = (1 - 1)\hat{i} + (2 - 1)\hat{j} + (1 - 0)\hat{k} = 0\hat{i} + 1\hat{j} + 1\hat{k}$$

$$\overrightarrow{AC} = (-2 - 1)\hat{i} + (2 - 1)\hat{j} + (-1 - 0)\hat{k} = -3\hat{i} + 1\hat{j} - 1\hat{k}$$

Normal vector $\vec{n} = \overrightarrow{AB} \times \overrightarrow{AC}$:

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & 1 \\ -3 & 1 & -1 \end{vmatrix}$$

$$= \hat{i}[(1)(-1) - (1)(1)] - \hat{j}[(0)(-1) - (1)(-3)] + \hat{k}[(0)(1) - (1)(-3)]$$

$$= \hat{i}[-1 - 1] - \hat{j}[0 + 3] + \hat{k}[0 + 3]$$

$$= -2\hat{i} - 3\hat{j} + 3\hat{k}$$

So direction ratios of normal are $-2, -3, 3$ (or equivalently $2, 3, -3$).

Equation of plane through $(1, 1, 0)$ with normal $(-2, -3, 3)$: $-2(x - 1) - 3(y - 1) + 3(z - 0) = 0$

$$-2x + 2 - 3y + 3 + 3z = 0$$

$$-2x - 3y + 3z + 5 = 0$$

$$2x + 3y - 3z = 5$$

Cartesian equation: $2x + 3y - 3z = 5$



Vector equation: $\vec{r} \cdot (2\hat{i} + 3\hat{j} - 3\hat{k}) = 5$

Q41. Show that the relation R defined in the set A of all triangles as $R \{(T_1, T_2): T_1 \text{ is similar to } T_2, \text{ where } T_1, T_2 \in A\}$ is an equivalence relation.

Answer: To prove R is an equivalence relation, we must show it is reflexive, symmetric, and transitive.

Reflexive: Every triangle T is similar to itself (since all corresponding angles are equal and all corresponding sides are in the same ratio 1:1). So $(T, T) \in R$ for all $T \in A$. **Hence R is reflexive.**

Symmetric: Let $(T_1, T_2) \in R$, i.e., T_1 is similar to T_2 . This means the corresponding angles of T_1 and T_2 are equal and corresponding sides are proportional. Since similarity is a mutual/reversible relationship (if $T_1 \sim T_2$ then $T_2 \sim T_1$), we get $(T_2, T_1) \in R$. **Hence R is symmetric.**

Transitive: Let $(T_1, T_2) \in R$ and $(T_2, T_3) \in R$, i.e., T_1 is similar to T_2 , and T_2 is similar to T_3 . Then the corresponding angles of T_1 equal those of T_2 , and the corresponding angles of T_2 equal those of T_3 . So the corresponding angles of T_1 equal those of T_3 , and similarly, the sides are proportional throughout. So T_1 is similar to T_3 , i.e., $(T_1, T_3) \in R$. **Hence R is transitive.**

Since R is reflexive, symmetric, and transitive, R is an equivalence relation.

Q42. Show that the tangents to the curve $6y = 3x^5 + 2x^3 - 3x$ at the points $x = \pm 3$ are parallel.

Answer: Differentiating $6y = 3x^5 + 2x^3 - 3x$ implicitly w.r.t. x : $6 \frac{dy}{dx} = 15x^4 + 6x^2 - 3$

$$\frac{dy}{dx} = \frac{15x^4 + 6x^2 - 3}{6}$$

Since the expression $15x^4 + 6x^2 - 3$ contains only even powers of x (i.e., x^4 and x^2), the slope $\frac{dy}{dx}$ is the same at $x = 3$ and $x = -3$ (since $(3)^4 = (-3)^4$ and $(3)^2 = (-3)^2$).

$$\text{At } x = 3: \frac{dy}{dx} = \frac{15(81) + 6(9) - 3}{6} = \frac{1215 + 54 - 3}{6} = \frac{1266}{6} = 211$$

$$\text{At } x = -3: \frac{dy}{dx} = \frac{15(81) + 6(9) - 3}{6} = \frac{1266}{6} = 211$$

Since the slopes at $x = 3$ and $x = -3$ are equal (both = 211), the tangents at these points are parallel.

Hence proved: the tangents are parallel.

OR

Find the intervals in which $f(x) = 2x^3 - 3x^2 - 12x + 6$ is increasing or decreasing.

Answer: $f'(x) = 6x^2 - 6x - 12 = 6(x^2 - x - 2)$

$$= 6(x - 2)(x + 1)$$

Setting $f'(x) = 0$: $x = 2$ or $x = -1$

These points divide the real line into three intervals: $(-\infty, -1)$, $(-1, 2)$, $(2, \infty)$.

Interval $(-\infty, -1)$: Take $x = -2$: $f'(-2) = 6(-2 - 2)(-2 + 1) = 6(-4)(-1) = 24 > 0$ increasing

Interval $(-1, 2)$: Take $x = 0$: $f'(0) = 6(0 - 2)(0 + 1) = 6(-2)(1) = -12 < 0$ decreasing



Interval $(2, \infty)$: Take $x = 3$: $f'(3) = 6(3 - 2)(3 + 1) = 6(1)(4) = 24 > 0$ increasing

- $f(x)$ is **increasing** on $(-\infty, -1) \cup (2, \infty)$
- $f(x)$ is **decreasing** on $(-1, 2)$

Q43. Show that the four points A, B, C and D with position vectors $4\hat{i} + 8\hat{j} + 12\hat{k}$, $2\hat{i} + 4\hat{j} + 6\hat{k}$, $3\hat{i} + 5\hat{j} + 4\hat{k}$ and $5\hat{i} + 8\hat{j} + 5\hat{k}$ respectively are coplanar.

Answer:

$$\vec{A} = 4\hat{i} + 8\hat{j} + 12\hat{k}, \vec{B} = 2\hat{i} + 4\hat{j} + 6\hat{k}, \vec{C} = 3\hat{i} + 5\hat{j} + 4\hat{k}, \vec{D} = 5\hat{i} + 8\hat{j} + 5\hat{k}$$

$$\vec{AB} = \vec{B} - \vec{A} = (2 - 4)\hat{i} + (4 - 8)\hat{j} + (6 - 12)\hat{k} = -2\hat{i} - 4\hat{j} - 6\hat{k}$$

$$\vec{AC} = \vec{C} - \vec{A} = (3 - 4)\hat{i} + (5 - 8)\hat{j} + (4 - 12)\hat{k} = -\hat{i} - 3\hat{j} - 8\hat{k}$$

$$\vec{AD} = \vec{D} - \vec{A} = (5 - 4)\hat{i} + (8 - 8)\hat{j} + (5 - 12)\hat{k} = \hat{i} + 0\hat{j} - 7\hat{k}$$

Four points A, B, C, D are coplanar if the scalar triple product $[\vec{AB} \vec{AC} \vec{AD}] = 0$

$$\begin{vmatrix} -2 & -4 & -6 \\ -1 & -3 & -8 \\ 1 & 0 & -7 \end{vmatrix}$$

Expanding along Row 1:

$$\begin{aligned} &= -2 \begin{vmatrix} -3 & -8 \\ 0 & -7 \end{vmatrix} - (-4) \begin{vmatrix} -1 & -8 \\ 1 & -7 \end{vmatrix} + (-6) \begin{vmatrix} -1 & -3 \\ 1 & 0 \end{vmatrix} \\ &= -2[(-3)(-7) - (-8)(0)] + 4[(-1)(-7) - (-8)(1)] - 6[(-1)(0) - (-3)(1)] \\ &= -2[21 - 0] + 4[7 + 8] - 6[0 + 3] \\ &= -2(21) + 4(15) - 6(3) \\ &= -42 + 60 - 18 \\ &= 0 \end{aligned}$$

Since the scalar triple product = 0, the vectors $\vec{AB}, \vec{AC}, \vec{AD}$ are coplanar.

Hence, the points A, B, C and D are coplanar.

Q44. Using the properties of determinants, prove that:
$$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$$

Answer: Let $\Delta = \begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$

Apply $R_1 \rightarrow R_1 + R_2 + R_3$

Row 1 becomes:

$$\begin{aligned} &(a - b - c + 2b + 2c), (2a + b - c - a + 2c), (2a + 2b + c - a - b) \\ &= (a + b + c), (a + b + c), (a + b + c) \end{aligned}$$



$$\Delta = \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

Taking $(a+b+c)$ common from Row 1: $\Delta = (a+b+c) \begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$

Apply $C_2 \rightarrow C_2 - C_1$ and $C_3 \rightarrow C_3 - C_1$:

Row 1: 1, 0, 0

Row 2: $2b$, $(b-c-a-2b)$, $(2b-2b) = 2b$, $(-b-c-a)$, 0

Row 3: $2c$, $(2c-2c)$, $(c-a-b-2c) = 2c$, 0, $(-a-b-c)$

$$\Delta = (a+b+c) \begin{vmatrix} 1 & 0 & 0 \\ 2b & -(a+b+c) & 0 \\ 2c & 0 & -(a+b+c) \end{vmatrix}$$

Expanding along Row 1 (since it has only one non-zero element, 1, in position (1,1)):

$$\begin{aligned} \Delta &= (a+b+c) \times 1 \times \begin{vmatrix} 0 & 0 \\ 0 & -(a+b+c) \end{vmatrix} \\ &= (a+b+c) \times [(-a-b-c) \times (-a-b-c) - 0] \\ &= (a+b+c) \times (a+b+c)^2 \\ &= (a+b+c)^3 \end{aligned}$$

Hence proved.

OR

If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$, then find A^{-1} . Using A^{-1} , solve the following system of linear equations:

$$x + y + z = -2$$

$$x + 2y - 3z = 12$$

$$2x - y + 3z = -13$$

Answer: Step 1: Find $|A|$

$$\begin{aligned} |A| &= 1(2 \times 3 - (-3)(-1)) - 1(1 \times 3 - (-3)(2)) + 1(1 \times (-1) - 2 \times 2) \\ &= 1(6 - 3) - 1(3 + 6) + 1(-1 - 4) \\ &= 1(3) - 1(9) + 1(-5) \\ &= 3 - 9 - 5 = -11 \end{aligned}$$

Step 2: Find the cofactors

$$C_{11} = +(2 \times 3 - (-3)(-1)) = 6 - 3 = 3$$

$$C_{12} = -(1 \times 3 - (-3)(2)) = -(3 + 6) = -9$$

$$C_{13} = +(1 \times (-1) - 2 \times 2) = -1 - 4 = -5$$

$$C_{21} = -(1 \times 3 - 1 \times (-1)) = -(3 + 1) = -4$$



$$C_{22} = +(1 \times 3 - 1 \times 2) = 3 - 2 = 1$$

$$C_{23} = -(1 \times (-1) - 1 \times 2) = -(-1 - 2) = 3$$

$$C_{31} = +(1 \times (-3) - 1 \times 2) = -3 - 2 = -5$$

$$C_{32} = -(1 \times (-3) - 1 \times 1) = -(-3 - 1) = 4$$

$$C_{33} = +(1 \times 2 - 1 \times 1) = 2 - 1 = 1$$

Step 3: Adjoint of A (transpose of cofactor matrix)

$$\text{adj}(A) = \begin{bmatrix} 3 & -4 & -5 \\ -9 & 1 & 4 \\ -5 & 3 & 1 \end{bmatrix}$$

Step 4: Inverse of A

$$A^{-1} = \frac{1}{|A|} \text{adj}(A) = \frac{1}{-11} \begin{bmatrix} 3 & -4 & -5 \\ -9 & 1 & 4 \\ -5 & 3 & 1 \end{bmatrix}$$

Step 5: Solve the system using $X = A^{-1}B$

The system in matrix form: $AX = B$, where $B = \begin{bmatrix} -2 \\ 12 \\ -13 \end{bmatrix}$

$$X = A^{-1}B = \frac{1}{-11} \begin{bmatrix} 3 & -4 & -5 \\ -9 & 1 & 4 \\ -5 & 3 & 1 \end{bmatrix} \begin{bmatrix} -2 \\ 12 \\ -13 \end{bmatrix}$$

Row 1: $3(-2) + (-4)(12) + (-5)(-13) = -6 - 48 + 65 = 113$

Row 2: $-9(-2) + 1(12) + 4(-13) = 18 + 12 - 52 = -22$

Row 3: $-5(-2) + 3(12) + 1(-13) = 10 + 36 - 13 = 33$

$$X = \frac{1}{-11} \begin{bmatrix} 11 \\ -22 \\ 33 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ -3 \end{bmatrix}$$

$$x = -1, y = 2, z = -3$$

Q45. Find the maximum and minimum values of $z = x + 2y$ subject to the following constraints:

$$x + 2y \geq 100, 2x - y \leq 0, 2x + y \leq 200, x \geq 0, y \geq 0$$

Answer: Constraint lines:

1. $x + 2y = 100 \rightarrow$ passes through $(100, 0)$ and $(0, 50)$
2. $2x - y = 0 \rightarrow$ passes through origin, i.e., $y = 2x$
3. $2x + y = 200 \rightarrow$ passes through $(100, 0)$ and $(0, 200)$

Finding corner points of the feasible region:

Point A: Intersection of $x + 2y = 100$ and $2x - y = 0$ (i.e., $y = 2x$)

$$x + 2(2x) = 100 \Rightarrow x + 4x = 100 \Rightarrow 5x = 100 \Rightarrow x = 20, y = 40$$



Point A = (20,40)

Point B: Intersection of $2x - y = 0$ and $2x + y = 200$

$$2x - y = 0 \Rightarrow y = 2x$$

$$2x + 2x = 200 \Rightarrow 4x = 200 \Rightarrow x = 50, y = 100$$

Point B = (50,100)

Point C: Intersection of $x + 2y = 100$ and x -axis ($y=0$)

$$x + 0 = 100 \Rightarrow x = 100$$

Point C = (100,0)

Point D: Intersection of $2x + y = 200$ and x -axis ($y=0$)

$$2x + 0 = 200 \Rightarrow x = 100$$

Point D = (100,0) same as Point C, so the feasible region touches the x -axis at (100, 0). So the corner points of the feasible (bounded) region are: (20, 40), (50, 100), and (100, 0)

Evaluating $z = x + 2y$ at each corner point:

$$\text{At } (20, 40): z = 20 + 2(40) = 20 + 80 = 100$$

$$\text{At } (50, 100): z = 50 + 2(100) = 50 + 200 = 250$$

$$\text{At } (100, 0): z = 100 + 2(0) = 100$$

Answer:

- **Maximum value of $z = 250$ at (50, 100)**
- **Minimum value of $z = 100$ at (20, 40) and also at (100, 0)**

OR

A company produces soft drinks that has a contract which requires that a minimum of 80 units of chemical A and 60 units of chemical B go into each bottle of the drink. The chemicals are available in prepared mix packets from two different suppliers, S and T. Supplier S had a packet of mix of 4 units of chemical A and Z units of chemical B and costs Rs 10. The supplier T had a packet of mix of 1 unit of A and 1 unit of B and costs Rs 4. Formulate this problem as an L.P.P so that the company honour the contract requirement at minimum cost. Also, obtain the feasible region corresponding to the constraints.

Answer: Formulation as L.P.P:

Objective function (minimize cost): Minimize $Z = 10x + 4y$

Constraints: For chemical A (minimum 80 units required): $4x + y \geq 80$

For chemical B (minimum 60 units required): $6x + y \geq 60$



Non-negativity constraints: $x \geq 0, y \geq 0$

Feasible Region:

To find the feasible region, plot the following lines:

Line 1: $4x + y = 80 \rightarrow$ passes through $(20, 0)$ and $(0, 80)$

Line 2: $6x + y = 60 \rightarrow$ passes through $(10, 0)$ and $(0, 60)$

Finding intersection of the two lines: $4x + y = 80 \dots (i)$

$6x + y = 60 \dots (ii)$

Subtracting (i) from (ii): $2x = -20 \Rightarrow x = -10$

Since $x = -10$ is negative, the two lines do not intersect within the first quadrant (feasible region).

This means one constraint dominates the other in the relevant region.

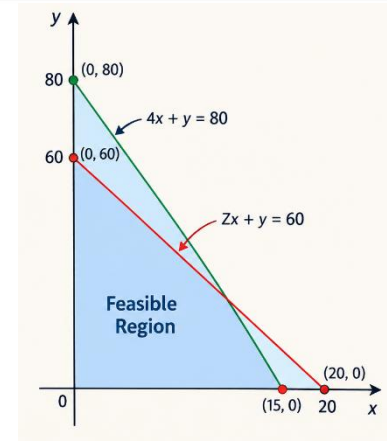
Testing which constraint is binding: At $x = 0$: Line 1 gives $y = 80$; Line 2 gives $y = 60$. Since we need $4x + y \geq 80$ AND $6x + y \geq 60$, and Line 1 requires the larger y -value at $x = 0$, the constraint $4x + y \geq 80$ is the more restrictive (binding) constraint near the y -axis.

The feasible region is the unbounded region in the first quadrant that lies above (or on) both lines $4x + y = 80$ and $6x + y = 60$ simultaneously, bounded by $x \geq 0$ and $y \geq 0$.

Since $4x + y \geq 80$ automatically satisfies $6x + y \geq 60$ when $x \geq 0$ (as shown by the non-intersection in the positive region and the boundary check above), the effective feasible region is determined mainly by: $4x + y \geq 80, x \geq 0, y \geq 0$

This is an unbounded feasible region in the first quadrant, lying on and above the line $4x + y = 80$.

(This completes the LPP formulation and the feasible region as required by the question the minimum cost point, if further required, would be found by evaluating $Z = 10x + 4y$ at the corner points of this unbounded feasible region, checking that the objective function doesn't decrease indefinitely by testing points beyond the corners.)





Thank you!



We hope you found this material helpful. We wish you the very best for your examination.



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