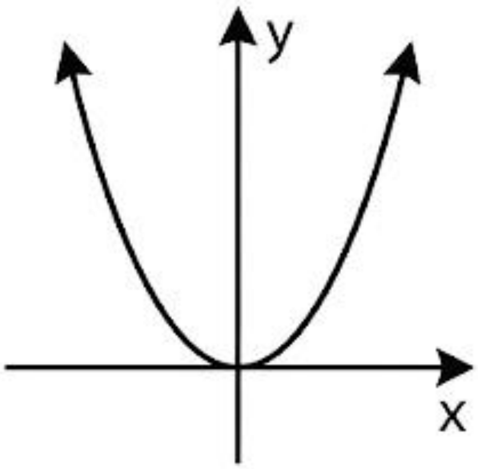
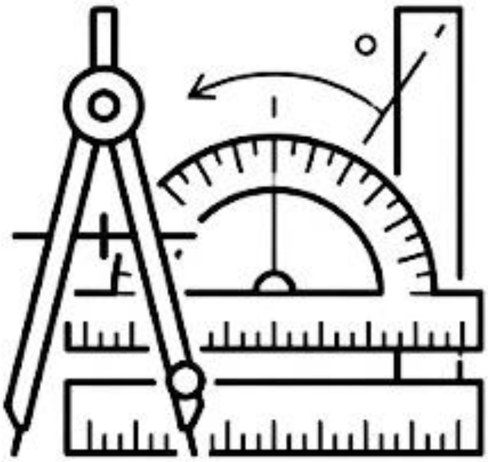
MVS FOUNDATION

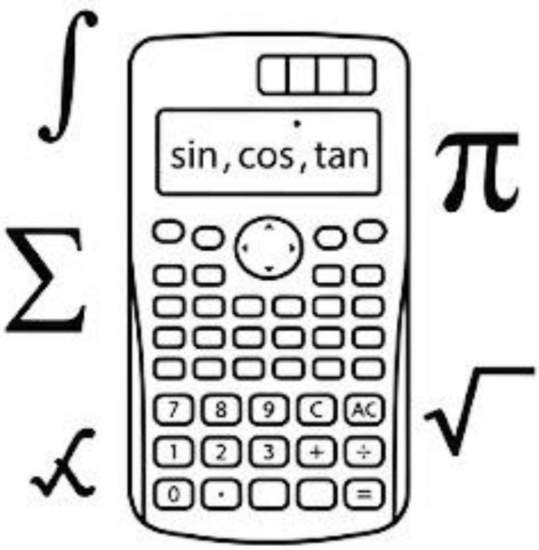
FREE NOTES

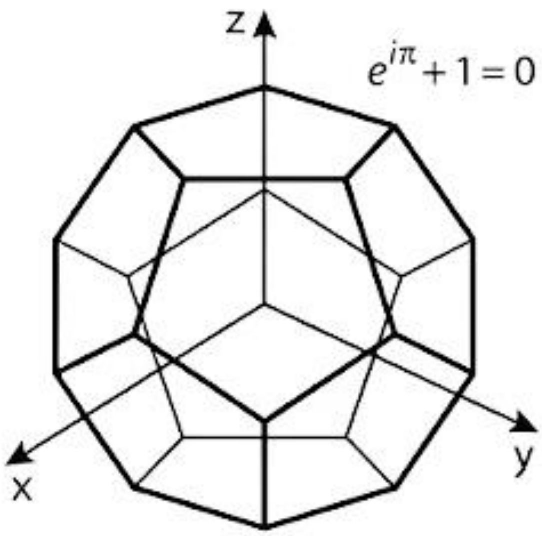
# MATHEMATICS (211)

## CHAPTERWISE NOTES





# MATHEMATICS

| S. No. | Module | Chapters (Public Examination) | Marks |
|---|---|---|---|
| **1** | Module 1: Algebra | **L-4: Special Product and Factorisation**<br>**L-5: Linear Equation**<br>**L-6: Quadratic Equation**<br>**L-7: Arithmetic Progression** | 20 |
| **2** | Module 2: Commercial Mathematics | **L-8: Percentage and its Applications**<br>**L-9: Installment Buying** | 8 |
| 3 | Module 6: Statistics | **L-25: Measure of Central Tendency L-26: Introduction to Probability** | 12 |

| Component | Details | Marks |
|---|---|---|
| **Public Exam (Selected Modules 1,2,6)** | Total Chapters: 8 | 40 |
| **Practical Exam** | Practical | 15 |
| **TMA** | Tutor Marked Assignment | 17 |
| **Final Possible Marks** | | **72 Marks** |

# Table of Contents

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# SPECIAL PRODUCTS AND FACTORIZATION

## Introduction

In this chapter, we will study the **special products** of polynomials and their **factorization**. With the help of special products, direct products can be found without actually writing all the terms of multiplication, which saves time and effort. We will also learn about the HCF, LCM of polynomials, and rational expressions.

## Special Products

Such products frequently used in algebra which make calculation shorter and easier are called **special products**. Important algebraic formulas (special products):

- $(a+b)^2 = a^2 + 2ab + b^2$
- $(a-b)^2 = a^2 - 2ab + b^2$
- $(a+b)(a-b) = a^2 - b^2$
- $(x+a)(x+b) = x^2 + (a+b)x + ab$
- $(ax+b)(cx+d) = acx^2 + (ad+bc)x + bd$

## Some Other Special Products

In this part, **special products** related to cubes of binomials are used.

Important formulas:

- $(a+b)^3 = a^3 + 3ab(a+b) + b^3$
- $(a-b)^3 = a^3 - 3ab(a-b) - b^3$
- $(a+b)(a^2 - ab + b^2) = a^3 + b^3$
- $(a-b)(a^2 + ab + b^2) = a^3 - b^3$

## Factorization of Polynomials

The process of writing a polynomial as a product of two or more polynomials is called **factorization**.

**(1) Factorization by Distributive Property**: Factorization is done by taking out a common term or variable present in all terms.

**(2) Factorization of a Perfect Square Trinomial**: In this, the formula $x^2 - y^2 = (x + y)(x - y)$is used.

**(3) Factorization of a Perfect Square Trinomial**: In this, the formula $x^2 + 2xy + y^2 = (x + y)^2$ or $x^2 - 2xy + y^2 = (x - y)^2$ is used.

**(4) Factorization of a Polynomial Reducible to the Difference of Two Squares**: By adding and subtracting an appropriate term, it is first made a perfect square, then the formula for the difference of squares is applied.

**(5) Factorization of Perfect Cube Polynomials**: In this, the perfect cube formula $x^3 + 3x^2y + 3xy^2 + y^3 = (x + y)^3$ is used.

**(6) Factorization of Polynomials Involving Sum or Difference of Two Cubes**: In this, the factorization formulae of $x^3 + y^3$ or $x^3 - y^3$ are used.

**(7) Factorizing Trinomials by Splitting the Middle Term**: In a polynomial of type $Ax^2 + Bx + C$ , the middle term B is divided into two parts whose sum is equal to B and product is equal to AC.

## HCF and LCM of Polynomials

**(1) HCF of Polynomials (HCF)**: It is the largest expression which is a common factor of all the given polynomials. It is found by the HCF of the common variables with minimum power and coefficients.

**(2) LCM of Polynomials (LCM)**: It is the smallest expression which is a multiple of each of the given polynomials. It is found by the LCM of all the variables with maximum power and coefficients.

## Rational Expressions

- The algebraic expression which is written in the form of $\frac{P}{Q}$ , where P and Q are polynomials and Q is a **non-zero polynomial**, is called a **rational expression**.
- Every polynomial is a rational expression, but it is not necessary that every rational expression is a polynomial.

## Operations on Rational Expressions

**(1) Addition and Subtraction of Rational Numbers**: The sum and difference of two rational expressions is always a rational expression. They are added or subtracted by taking the LCM of the denominator.

**(2) Multiplication and Division of Rational Expressions:**

- Formula for multiplication: $\frac{P}{Q} \times \frac{R}{S} = \frac{PR}{QS}$
- Formula for division: $\frac{P}{Q} \div \frac{R}{S} = \frac{P}{Q} \times \frac{S}{R}$ (For division, we multiply by taking the reciprocal of the second expression).

# TOP 5 QUESTIONS

**Q1. Find the factors of $3x^2 - x - 4$.**

**Answer-** On splitting the middle term: $3x^2 - 4x + 3x - 4$

$= x(3x - 4) + 1(3x - 4)$

$= (3x - 4)(x + 1)$

**Q2. If $x - \frac{1}{x} = 2$ , then find the value of $x^2 + \frac{1}{x^2}$ .**

**Answer-** On squaring both sides: $\left(x - \frac{1}{x}\right)^2 = (2)^2$

$\Rightarrow x^2 + \frac{1}{x^2} - 2(x)\left(\frac{1}{x}\right) = 4$

$\Rightarrow x^2 + \frac{1}{x^2} - 2 = 4$

$\Rightarrow x^2 + \frac{1}{x^2} = 6$ I

**Q3. Find the LCM of $P(x) = (x - 2)(x^2 - 3x + 2)$ and $Q(x) = x^2 - 4$.**

**Answer- Factors of $P(x)$:** $(x - 2)(x - 2)(x - 1) = (x - 2)^2(x - 1)$

**Factors of $Q(x)$:** $(x - 2)(x + 2)$

For LCM, the maximum power of all variables is taken: $\mathbf{LCM} = \mathbf{(x - 2)^2(x + 2)(x - 1)}$

**Q4. What will be the product of $(x + 5)(x - 5)$?**

**Answer-** Here the formula $(\boldsymbol{a} + \boldsymbol{b})(\boldsymbol{a} - \boldsymbol{b}) = \boldsymbol{a}^2 - \boldsymbol{b}^2$ will be used.

On substituting $a = x$ and $b = 5$: $(x)^2 - (5)^2 = x^2 - 25$ is obtained.

**Q5. Factorize $x^2 + 6x + 9$ by the perfect square method.**

**Answer-** This can be written in the form of the formula $\boldsymbol{a}^2 + 2\boldsymbol{ab} + \boldsymbol{b}^2 = (\boldsymbol{a} + \boldsymbol{b})^2$.

Here $a = x$ and $b = 3$. Therefore, $x^2 + 2(x)(3) + (3)^2 = (x + 3)^2$.

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# Linear Equations

## Introduction

In this chapter, we will study the concept of **linear equations** in one variable and two variables, their formation, and the methods to solve them. We will also learn to find their solutions by converting word problems of daily life into equations through algebraic and graphical methods.

## Linear Equations

- The equation in which the sign of equality (equal '=') is present and the maximum power of the variable is 1, is called a **linear equation**.
- The sign of equality shows that the left-hand side (LHS) and the right-hand side (RHS) are equal to each other.
- The general form of a linear equation in one variable: $ax + b = 0$ (where a $\neq$ 0 and a, b are constants).
- The general form of a linear equation in two variables: $ax + by + c = 0$ where a and b both cannot be zero simultaneously).

## Formation of a Linear Equation in One Variable

- To give a mathematical form to word (daily life) problems, by assuming the unknown quantity as a variable (like x or y), an equation is formed according to the given conditions.

## Solution of Linear Equations in One Variable

- That value of the variable which, when substituted in the equation, makes LHS and RHS equal, is called the **solution** of the equation.
- To maintain the balance of the equation, the same number can be added, subtracted, multiplied or divided on both sides.
- The process of moving terms from one side to the other side is called **transposition**, in which the sign of the term changes ('+' to '-' and '-' to '+').

- **Solution in formula form:** The solution of the equation $ax + b = 0$ is $x = -\frac{b}{a}$.

**Word Problems**

- By reading real life problems related to age, numbers, perimeter etc., assuming the unknown quantity as 'x', it is converted into a linear equation in one variable and then solved.

## Linear Equations in Two Variables

- Linear equations in two variables ($ax + by + c = 0$) have **infinite (countless) solutions**.
- For every value of y, a unique solution of x is obtained, which can be found by this formula:

$x = -\frac{b}{a}y - \frac{c}{a}$.

## Graph of Linear Equations in Two Variables

- The graph of a linear equation in two variables is always a **straight line**.
- To draw the graph, by finding at least two values of x and y (by forming ordered pairs) on the Cartesian plane (x-axis and y-axis), they are plotted and joined.
- Whatever points (coordinates) lie on this line, they are the solutions of the equation.

## System of Linear Equations in Two Variables

When two linear equations in two variables are taken together, they are called a **system** of equations.

### Graphical Method:

- **Intersecting lines**: Both graphs intersect at one point. The system has **a unique solution**.
- **Coincident lines**: Both graphs are formed on the same line (one over the other). The system has **infinite solutions**.
- **Parallel lines**: Both graphs never intersect each other. The system has **no solution**.

### Algebraic Method:

**(i) Substitution Method**: By finding the value of one variable (like x) from one equation, it is **substituted** in the other equation, thereby making it an equation of one variable.

**(ii) Elimination Method**: By multiplying both equations by some number, the coefficients of any one variable are made equal. Then by adding or subtracting, that variable is **eliminated** (disappears).

**Word Problems**

- Problems involving two unknown quantities (like the speed of two cars, the price of two items, fractions etc.) are formed into a system of two equations by assuming x and y, and then solved by the algebraic method.

# TOP 5 QUESTIONS

**Q1. Find the coefficient of $y$ in the linear equation $5(2x-4)+3x+4y-7=0$.**

**Answer-** On solving the equation:

$10x - 20 + 3x + 4y - 7 = 0$

$\Rightarrow 13x + 4y - 27 = 0$l

Here 4 is being multiplied with y, therefore **the coefficient of y is 4**.

**Q2. If $y=3$ is expressed in the form $ax+by+c=0$, then what will be the value of $a$?**

**Answer-** We can write $y = 3\ as\ 0 \cdot x + 1 \cdot y - 3 = 0$.

On comparing with the general form $ax+by+c=0$, the coefficient of $x$ obtained is $a = 0$.

**Q3. At which point does the graph of the equation $2x - 3y = 6$ intersect the y-axis?**

**Answer-** On the y-axis, x = 0. On substituting $x = 0$ in the equation:

$2(0) - 3y = 6 \Rightarrow -3y = 6 \Rightarrow y = -2$

Therefore, the intersection point is $(0, -2)$.

**Q4. Solve the system of equations $x+y=5$ and $x-y=1$ by substitution or elimination method.**

**Answer-** Elimination method (on adding both):

$(x+y) + (x-y) = 5+1 \Rightarrow 2x = 6 \Rightarrow x = 3$

On substituting $x = 3$ in equation: $3 + y = 5 \Rightarrow y = 2$. Solution: $x = 3, y = 2$.

**Q5. The sum of two numbers is 15 and their difference is 3. Form linear equations and find the numbers.**

**Answer-** Let the numbers be x and y.

Equations: $x + y = 15$ and $x - y = 3$.

On adding both $2x = 18 \Rightarrow x = 9$.

On substituting the value of $x$, $9 + y = 15 \Rightarrow y = 6$. The numbers are **9 and 6**.

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# Quadratic Equations

## Introduction

In this chapter, we will study **quadratic equations** and various methods to solve them. We will learn how an equation is formed by equating a quadratic polynomial to zero. Besides this, we will also learn to solve word problems of real life using the factorization method and the quadratic formula.

## Quadratic Equation

- A polynomial of degree two is called a **quadratic polynomial**.
- When a quadratic polynomial is equated to zero, it is called a **quadratic equation**.

## Standard Form of a Quadratic Equation

- A quadratic equation $ax^2 + bx + c = 0$ (where $a \neq 0$ and $a, b, c$ are constants and x is a variable) is called the **standard form** of a quadratic equation.
- Every quadratic equation can always be expressed in standard form (like $3x^2 = 5$ can be written as $3x^2 - 5 = 0$).

## Solution of a Quadratic Equation

- That value of the variable, which when substituted in the left and right side of the equation makes both sides equal, is called the **root or solution** of the quadratic equation.
- There are two algebraic methods to find the solution of a quadratic equation: factorization method and quadratic formula.

## Factorization Method

- In this method, the quadratic equation is expressed as the product of linear factors (usually by splitting the middle term).
- The roots of the equation (values of x) are found by equating each linear factor to zero.

## Quadratic Formula

1. The **quadratic formula** to find the roots of the quadratic equation $ax^2 + bx + c = 0$ is as follows:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

2. The expression $b^2 - 4ac$ is called the **Discriminant** (D), which tells the nature of the roots of the equation:

- If **D > 0**: The equation will have two real and distinct roots.
- If **D = 0**: The equation will have two real and equal roots (each $-\frac{b}{2a}$).
- If **D < 0**: The equation will have no real roots (because the square root of a negative number is not real).

## Word Problems

- Problems of daily life related to numbers, age, area etc. are first converted into mathematical language (quadratic equation).
- After this, the value of the unknown quantity is found by solving that quadratic equation using the factorization method or the quadratic formula.

# TOP 5 QUESTIONS

**Q1. Check whether $x + \frac{1}{x} = \frac{5}{2}$ is a quadratic equation?**

**Answer-** On simplifying it:

$$\frac{x^2 + 1}{x} = \frac{5}{2} \Rightarrow 2(x^2 + 1) = 5x \Rightarrow 2x^2 - 5x + 2 = 0$$

is obtained.

Since it is in the standard form of $ax^2 + bx + c = 0$, therefore it is a quadratic equation.

**Q2. Solve the equation $6x^2 + 7x - 3 = 0$ by factorization method.**

**Answer-** On splitting the middle term: $6x^2 + 9x - 2x - 3 = 0$

$\Rightarrow 3x(2x + 3) - 1(2x + 3) = 0$

$\Rightarrow (2x + 3)(3x - 1) = 0$

Therefore, $x = -\frac{3}{2}$ and $x = \frac{1}{3}$ are the solutions of the equation.

**Q3. Find the roots of the equation $6x^2 - 19x + 15 = 0$ by quadratic formula.**

**Answer-** Here $a = 6, b = -19, c = 15$.

**Discriminant $D$** $= (-19)^2 - 4(6)(15) = 361 - 360 = 1$

**From formula $x = \frac{-b \pm \sqrt{D}}{2a}$**

$x = \frac{19 \pm 1}{12}$

Therefore, roots are $\frac{5}{3}$ and $\frac{3}{2}$.

**Q4. Find the value of m for which the roots of the equation $2x^2 - mx + 1 = 0$ are equal.**

**Answer-** For equal roots $D = 0 \Rightarrow b^2 - 4ac = 0$ should be there.

Here $a = 2, b = -m, c = 1$.

Therefore, $(-m)^2 - 4(2)(1) = 0 \Rightarrow m^2 - 8 = 0 \Rightarrow m = \pm 2\sqrt{2}$

**Q5. The sum of the squares of two consecutive odd natural numbers is 74. Find the numbers.**

**Answer-** Let the numbers be $x$ and $x + 2$.

$x^2 + (x+2)^2 = 74 \Rightarrow 2x^2 + 4x - 70 = 0 \Rightarrow x^2 + 2x - 35 = 0$

Factors: $(x+7)(x-5) = 0$. Natural numbers are not negative, therefore $x = 5$. The numbers are 5 and 7.

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# Arithmetic Progression

## Introduction

In nature and our daily life, many objects follow a special pattern. In this chapter, we will study one such special number pattern which is called **arithmetic progression**. We will learn the formulae and methods to find its general term (nth term) and the sum of its first n terms.

## Some Number Patterns

- Each number in the list of numbers is called a **term**.
- The numbers in the list are generally denoted by $a_1, a_2, a_3, \dots, a_n$ $or$ $t_1, t_2, t_3, \dots, t_n$ which are called the first, second and nth term respectively.
- These lists are also called **sequences** or **number patterns**.

## Arithmetic Progression

- A special type of pattern, in which except the first term, each term is obtained by adding a fixed amount (positive or negative) to its preceding term, is called an **Arithmetic Progression (AP)**.
- The first term is usually denoted by **'a'** and the fixed amount (common difference) is denoted by **'d'**.
- The **standard form** of an arithmetic progression is $a, a + d, a + 2d, a + 3d, \dots$

## General (nth) Term of an Arithmetic Progression

- If the first term of an arithmetic progression is a and the common difference is d, then the formula to find its **general term** (nth term) is as follows:
- **Formula:** $t_n = a + (n - 1)d$
- Here n denotes the number of terms.

## Sum of the First n Terms of an Arithmetic Progression

- To find the sum of the first n terms ($S_n$) of an arithmetic progression, this formula is used:

- **Formula:** $\boldsymbol{S_n = \frac{n}{2}[2a + (n-1)d]}$

- If the first term of the arithmetic progression is $a$, the last term is $l$ (or $t_n$) and the number of terms is $n$, then another formula for the sum is this:

- **Alternative Formula:** $\boldsymbol{S_n = \frac{n}{2}(a + 1)}$

# TOP 5 QUESTIONS

**Q1. Find the 15th term of the arithmetic progression $\boldsymbol{16, 11, 6, 1, -4, \ldots}$**

**Answer-** Here $a = 16$ and $d = 11 - 16 = -5$

On using the formula $t_n = a + (n-1)d$

$t_{15} = 16 + (15-1)(-5) = 16 + 14(-5) = 16 - 70 = -54$ I

**Q2. The common difference of an AP is 5 and the 10th term is 43. Find its first term.**

**Answer-** Here $d = 5$ and $t_{10} = 43$.

On substituting the values in the formula $t_{10} = a + 9d$

$43 = a + 9(5) \Rightarrow 43 = a + 45 \Rightarrow a = -2$

Therefore, the first term is $\boldsymbol{-2}$.

**Q3 Find the sum of the first 12 terms of the arithmetic progression $\boldsymbol{11, 16, 21, 26 \ldots}$.**

**Answer-** Here $a = 11$, $d = 16 - 11 = 5$ and $n = 12$.

From the formula $S_n = \frac{n}{2}[2a + (n-1)d]$:

$S_{12} = \frac{12}{2}[2(11) + (11)5] = 6[22 + 55] = 6 \times 77 = 462$

**Q4. The sum of how many terms of the arithmetic progression $2, 4, 6, 8, 10....$ will be 210?**

**Answer-** Here $a = 2, d = 2$ and $S_n = 210$.

On substituting in $S_n = \frac{n}{2}[2a + (n-1)d]$.

$210 = \frac{n}{2}[4 + (n-1)2] \Rightarrow 420 = n(2n+2) \Rightarrow n^2 + n - 210 = 0$

On solving $n = 14$ (negative value is invalid).

**Q5. Find the sum: $2 + 5 + 8 + 11 + .... + 59$.**

**Answer-** Here $a = 2, d = 3$ and $t_n = 59$.

First find n: $59 = 2 + (n-1)3 \Rightarrow 57 = 3(n-1) \Rightarrow n = 20$

Now the sum $S_{20} = [2 + 59] = 10 \times 61 = 610$

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# Percentage and its Applications

## Introduction

In this chapter, we will study the concept of **percent** (meaning 'per hundred') and the methods to convert it into a fraction or a decimal. Along with this, we will learn to calculate its practical applications like profit-loss, discount, simple interest, compound interest, and growth and depreciation.

## Percent

- The word **percent** is derived from the Latin word 'per centum', which means **"per hundred"** or out of "a hundred".
- A fraction whose denominator is 100 is read as a percent. The symbol **'%'** is used to denote it.

## Conversion of a Fraction into Percent and vice versa

- **Converting a fraction into percent**: Multiply the fraction by 100, simplify it, and put the **%** sign.
- **Converting a percent into fraction:** Remove the % sign and multiply the number by $\frac{1}{100}$ (or divide by 100) and simplify.

## Conversion of Decimal into a Percent and vice versa

- **Converting a decimal into percent**: Shift the decimal point two places to the right and put the **%** sign.
- **Converting a percent into decimal**: Remove the **%** sign and shift the decimal point two places to the left.

## Calculation of percent of a quantity or a number

- To find a specified percentage of a quantity, convert the percent into a fraction or decimal and then multiply it by the given number.
- **Formula**: $x\ of\ y\% = x \times \left(\frac{y}{100}\right)$

# Application of Percentage

## Profit and Loss

**1. Cost Price (C.P.)**: The price at which an article is bought.

**2. Selling Price (S.P.)**: The price at which an article is sold.

**3. Profit**: When S.P. > C.P. **Formula**: Profit = S.P. – C.P.

**4. Loss**: When C.P. > S.P. **Formula**: Loss = C.P. – S.P.

Important formulas:

- $\textbf{Profit}\ \% = \left(\frac{\textbf{Profit}}{\textbf{C.P.}}\right) \times \textbf{100\%}$
- $\textbf{Loss}\ \% = \left(\frac{\textbf{Loss}}{\textbf{C.P.}}\right) \times \textbf{100\%}$
- $\textbf{S. P.} = \frac{\textbf{C.P.} \times (\textbf{100} + \textbf{Profit}\ \%)}{\textbf{100}}$
- वि. मू. $= \frac{\textbf{C.P.} \times (\textbf{100} - \textbf{Loss}\ \%)}{\textbf{100}}$

## Discount

- **Discount**: It is the reduction given on the marked price (list price) of an article.
- **Marked Price**: The price written or printed on an article.
- **Formula**: $\text{Actual Selling Price} = \text{Marked Price} - \text{Discount}$

## Simple Interest

- **Principal (P)**: The money borrowed or lent.
- **Interest (I)**: The additional money paid for using the borrowed money.
- **Amount (A)**: The sum of the principal and interest. $A = P + I$
- **Formula for Simple Interest: I** $= \frac{\textbf{P} \times \textbf{R} \times \textbf{T}}{\textbf{100}}$ (Where R = Rate % per annum, T = Time)

## Compound Interest

- When after a specified period, the interest is added to the principal and the interest for the next period is calculated on this new principal, it is called **compound interest**.
- The period for adding the interest is called the **conversion period**.

**Formula for compound interest**

- **Formula for Amount (A):** $\mathbf{A = P\left(1+\frac{R}{100}\right)^{n}}$ (Where n = number of periods/years)
- **Formula for Compound Interest (CI):** $\mathbf{CI = A - P = P\left[\left(1+\frac{R}{100}\right)^{n} - 1\right]}$

## Rate of Growth and Depreciation

- If the initial value is $V_o$, rate is $r\%$ and time is $n$, then:
- **Formula for Growth:** $V_n = V_o\left(1+\frac{r}{100}\right)^{n}$
- **Formula for Depreciation:** $V_n = V_o\left(1-\frac{r}{100}\right)^{n}$

# TOP 5 QUESTIONS

**Q1. 144 is what percent of 360?**

**Answer-** Let $x\%\ \boldsymbol{of}\ \mathbf{360 = 144}$

Therefore, $\frac{x}{100} \times 360 = 144$

On solving this, $x = \frac{144 \times 100}{360} = 40$ is obtained.

Therefore, 40% of 360 is 144.

**Q2. A shopkeeper buys an article for ₹360 and sells it for ₹270. Find his profit or loss percent.**

**Answer-** Here **C.P. = ₹360** and **S.P. = ₹270**.

Since **C.P. > S.P.,** therefore loss = $\mathbf{360 - 270 = ₹90}$ occurred.

**Loss percent** = $\frac{90}{360} \times 100\% = 25\%$

**Q3. The marked price of a coat is ₹2400. Find its selling price if a discount of 12% is being given.**

**Answer- Discount given** = $12\%\ of\ 2400 = 2400 \times \frac{12}{100} =$ ₹288

**Actual selling price** = $Marked\ Price - Discount = 2400 - 288 =$ ₹2112

**Q4. At what rate of simple interest per annum will a sum of ₹5000 amount to ₹6050 in 3 years?**

**Answer-** Here Principal (P) = ₹5000, Amount (A) = ₹6050, Time (T) = 3 years.

$\boldsymbol{Interest\ (I)} = 6050 - 5000 =$ ₹1050

$\boldsymbol{Rate\ R} = \frac{(I \times 100)}{(P \times T)} = \frac{1050 \times 100}{5000 \times 3} = 7\%$ per annum.

**Q5. Find the compound interest on ₹20,000 for 3 years at the rate of 5% per annum when the interest is compounded annually.**

**Answer-** From the formula $A = P\left(1 + \frac{R}{100}\right)^n$,

$Amount\ A = 20000\left(1 + \frac{5}{100}\right)^3 = 20000\left(\frac{21}{20}\right)^3 =$ ₹23152.50

$Compound\ interest = 23152.50 - 20000 =$ ₹3152.50

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# Instalment Buying

## Introduction

In this chapter, we will study about the **Instalment Plan**. We will learn how expensive articles can be purchased by making a partial payment (cash down payment) and paying the remaining amount in instalments. Under this, we will learn to calculate the instalment amount, rate of interest, and cash price on the basis of simple and compound interest.

## Instalment Buying Scheme-Some Definitions

- **Cash Price**: This is the amount at which an article can be purchased if the entire money is paid at once.
- **Cash Down Payment**: This is the partial amount which is paid in cash by the customer at the time of purchasing the article.
- **Installment**: This is the amount which is paid by the customer at regular intervals of time (monthly, half-yearly, or yearly) to pay off the remaining price.
- **Interest under Instalment Plan**: Since the payment of the remaining amount is done later, the seller charges some extra money, which is called **interest**.

## To Find the Interest in an Instalment Plan

When the cash price, cash down payment, number of instalments, and amount of instalment are given, then we can find the rate of interest (r) using the simple interest formula.

**Method**:

- Total interest paid = $(Cash\ down\ payment\ +\ Amount\ of\ all\ instalments)\ -\ Cash\ price.$
- By finding the remaining amount for each month, the total **principal** for 1 month is determined.
- **Formula**: Interest $= \frac{\text{Total Principal} \times r \times 1}{100 \times 12}$

## To Find the Amount of Instalment

When the cash price, cash down payment, rate of interest, and number of instalments are given, then the amount of each instalment is determined.

**Method**:

- Let the amount of each instalment be $x$.
- By expressing the total interest and total principal in terms of $x$, they are put in the simple interest formula and $x$ is found by solving the equation.

## To Find the Cash Price

When the cash down payment, amount of instalment, rate of interest, and number of instalments are given, then the **cash price** of the article is determined.

**Method**:

- Let the cash price be $x$.
- By writing the given interest and total principal in terms of $x$, the equation is solved using the simple interest formula.

## Problems Involving Compound Interest

When instalments are given for a longer period (like yearly or half-yearly), then instead of simple interest, **compound interest** is used there.

**Method**:

- The **Present Value (P)** of each instalment is determined.
- **Formula**: If the rate of interest is $R\%$ and the instalment is $x$, then the present value $P_n$ of the n$^{th}$ instalment will be as follows: $x = P_n\left(1 + \frac{R}{100}\right)^n$
- Sum of the present values of all instalments

  $(P_1 + P_2 + \cdots) = \text{Remaining amount to be paid (Cash price} - \text{Cash down payment)}.$

# TOP 5 QUESTIONS

**Q1. A table is sold for ₹450 cash or for ₹210 cash down payment followed by two monthly instalments of ₹125 each. Find the rate of interest charged under the instalment plan.**

**Answer-** Total payment $= 210 + (125 \times 2) = ₹460$

Interest $= 460 - 450 = ₹10$

Principal for the first month = ₹240, for the second month = ₹115

Total principal = ₹355

On solving the formula = $\frac{355\times r\times 1}{100\times 12} = 10$

$\mathbf{r = 33.8}\%$ per annum.

**Q2. The cash price of a ceiling fan is ₹1940. It is available for ₹420 cash down payment and three equal monthly instalments. If the interest is 16% per annum, then find the amount of each instalment.**

**Answer-** Let the instalment be $x$.

Total payment $= 420 + 3x$

Interest $= 3x - 1520$

Total principal for 1 month $= (4560 - 3x)$

On solving the equation $(3x - 1520) = \frac{(4560 - 3x)\times 16\times 1}{100\times 12}$

$\mathbf{x = ₹520}$

**Q3. A mixie was purchased for an immediate payment of ₹360 and three equal monthly instalments of ₹390 each. If the rate of interest is 16% per annum, then find the cash price of the mixie.**

**Answer-** Let the cash price be $x$.

Total payment $= 360 + 1170 = ₹1530$

Interest $= 1530 - x$

Total principal $= 3x - 2250$

On solving $(1530 - x) = \frac{(3x - 2250) \times 16 \times 1}{100 \times 12}$

the cash price $\mathbf{x} = \mathbf{₹1500}$

**Q4. A refrigerator is available for ₹12000 cash or for a cash down payment of ₹3600 and 2 equal half-yearly instalments. If the rate is 20% per annum (compounded half-yearly), then the amount of the instalment?**

**Answer-** Remaining amount $= 12000 - 3600 = ₹8400$

$Rate = 10\%\ per\ half\ year.$

From the compound interest formula: $\frac{10}{11}x + \frac{100}{121}x = 8400$

On solving, each instalment $\mathbf{x} = \mathbf{₹4840}$

**Q5. A sewing machine is available for ₹2600 cash or for a cash down payment of ₹1000 and three equal monthly instalments of ₹550. Find the rate of interest.**

**Answer-** $Total\ payment = 1000 + (550 \times 3) = ₹2650$

Interest $= 2650 - 2600 = ₹50$

Principal for the first month = 1600, for the second = 1050, for the third = 500

Total principal = ₹3150

$\frac{3150 \times r \times 1}{100 \times 12} = 50 \Rightarrow \mathbf{r} = \mathbf{19\frac{1}{21}\%\ per\ annum.}$

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# Measures of Central Tendency

## Introduction

In this chapter, we will study the **measures of central tendency** of data under statistics. The average or central value that represents a large group of data is called the measure of central tendency. We will learn the methods and their formulae to find the mean, median, and mode for ungrouped and grouped data.

## Arithmetic Average or Mean

- **Mean** is the value which is obtained by dividing the sum of all the observations by their total number.
- It is represented mathematically by $\bar{x}$ (x bar).

## Mean of Raw Data

- To find the mean of ungrouped (raw) data, all the observations are added and divided by the total number.
- **Formula:**

$$\bar{x} = \frac{\Sigma x_i}{n}$$

(where n is the total number of observations).

## Mean of Ungrouped Data

- When data is given with frequencies $(f_i)$, the sum is found by multiplying each observation $(x_i)$ by its corresponding frequency.
- **Formula:** $\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}$

## Mean of Grouped Data

There are three main methods to find the mean in grouped data (class intervals):

**(i) Direct Method:** In this, the class mark $(x_i)$ is found and multiplied by $f_i$.

- **Formula:** $\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}$

**(ii) Assumed Mean Method:** In this, any middle value is assumed as the **Assumed Mean (A)** and the deviation $d_i = x_i - A$ is found.

- **Formula:** $\bar{x} = A + \frac{\Sigma f_i d_i}{\Sigma f_i}$

**(iii) Step Deviation Method:** To make the calculation easier, the deviation $d_i$ is divided by the class size $(h)$ to find $u_i$.

- **Formula:** $\bar{x} = A + h \times \left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right)$

## Median

- **Median** is the value that divides the data into exactly two equal parts when they are arranged in an ascending (increasing) or descending (decreasing) order.

## Median of Raw Data

**1.** First of all, the data is written in ascending order.

**2.** If the number of terms (n) is **Odd**, then:

- **Formula:** $\text{Median} = \left(\frac{n+1}{2}\right)\text{th term}$

**3.** If the number of terms (n) is **Even**, then:

- **Formula:** $\text{Median} = \frac{\left(\frac{n}{2}\right)\text{th term} + \left(\frac{n}{2}+1\right)\text{th term}}{2}$

## Median of Ungrouped Data

- In this, first of all, the **Cumulative Frequency (cf)** is found.
- Half of the total frequency $(N = \sum f_i)$, which is $\left(\frac{N}{2}\right)$, is calculated.
- The observation whose $cf$ is just greater than $\frac{N}{2}$, is called the median.

## Mode

- **Mode** is the value whose frequency is the highest, that is, which is repeated the most number of times in the data.

## Mode of Raw Data

- In raw data, it is just observed which number has appeared the most number of times. That number is the mode.

## Mode of Ungrouped Data

- In the given frequency table, the observation ($x_i$) in front of which the frequency ($f_i$) is the largest (maximum), that observation is the mode.

# TOP 5 QUESTIONS

**Q1Find the mean of the first 5 natural numbers.**

**Answer-** The first 5 natural numbers are: 1, 2, 3, 4, 5.

$$Mean = \frac{Sum\ of\ all\ observations}{Total\ Numbers}$$

$$= \frac{1+2+3+4+5}{5} = \frac{15}{5} = 3$$

Therefore, the mean is **3**.

**Q2. Find the mode of the data 15, 14, 19, 20, 14, 15, 14, 18.**

**Answer-** In the given data, the number '14' has appeared the most number of times (three times).

Since the observation whose frequency is the highest is the mode, therefore the mode of this data is **14**.

**Q3. Find the median of the data 3, 5, 7, 9, 11, 13, 15.**

**Answer-** The data is already in ascending order. Here the number of terms is $n = 7$ (Odd).

$$Median = \frac{n+1}{2}\ th\ term \Rightarrow \frac{7+1}{2} = 4\ th\ term.$$

The 4th term in the data is 9, therefore the median is 9.

**Q4. If the mean of 10 observations is 15, then find the total sum of all the observations.**

**Answer- According to the formula:** $Mean = \frac{Sum\ of\ observations}{Numbers\ of\ observations}$

Here Mean = 15 and number = 10.

Therefore, sum of observations = $Mean \times number = 15 \times 10 = \mathbf{150}$.

**Q5. Find the median of the data 2, 4, 6, 8, 10, 12.**

**Answer-** Here the data is in ascending order and the number of terms is $n = 6$ (Even).

$$Median = \frac{\left(\frac{6}{2}\right)th\ term + \left(\frac{6}{2}+1\right)th\ term}{2}$$

$$= \frac{3rd\ term + 4th\ term}{2} = \frac{6+8}{2} = \frac{14}{2} = \mathbf{7}$$

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# Introduction to Probability

## Introduction to Probability

In this chapter, we will study **Probability**, which is a method to measure the situations of uncertainty in mathematical form. We will learn random experiments, their outcomes, events, and the important formulae to calculate the probability of occurrence or non-occurrence of an event.

## Random Experiment

- That experiment whose all possible outcomes are known in advance, but the exact prediction of any particular outcome cannot be made, is called a **random experiment**.

## Outcomes and Sample Space

- **Outcome:** Each possible result of a random experiment is called an outcome.
- **Sample Space:** The collection of all possible outcomes of an experiment is called the sample space.

## Event and its Probability

- **Event:** Any particular collection or part of the outcomes of an experiment is called an event.
- The **probability P(E)** of occurrence of an event $E$ is defined as the ratio of favorable outcomes and total possible outcomes.
- **Formula for Probability:** $\mathrm{P(E)} = \dfrac{\textbf{Number of outcomes favourable to E}}{\textbf{Total number of possible outcomes}}$
- The probability of any event always lies between 0 and 1: $0 \leq P(E) \leq 1$

## Sure and Impossible Events

- **Sure Event:** That event whose occurrence is completely certain. Its probability is always **1**.
- **Impossible Event:** That event whose occurrence is not possible. Its probability is always **0**.

## Complementary Events

- The event of 'not occurring' of an event E is called its **complementary event**, which is denoted by $\bar{E}$ (E bar).
- The total sum of the probabilities of occurrence and non-occurrence of an event is always 1.
- **Formula:** $P(E) + (P(\bar{E}) = 1 \; or \; P(\bar{E}) = 1 - P(E)$

# TOP 5 QUESTIONS

**Q1. If the probability of occurrence of an event E is $P(E)$ = 0.35, then what will be the probability $P(\bar{E})$ of 'not occurring' of E?**

**Answer-** On using the formula $P(E) + P(\bar{E}) = 1$:

$0.35 + P(\bar{E}) = 1$

$P(\bar{E}) = 1 - 0.35 = 0.65$

Therefore, the probability of non-occurrence of the event is **0.65**.

**Q2. A dice is thrown once. Find the probability of getting a prime number.**

**Answer-** Total possible outcomes of the dice (1, 2, 3, 4, 5, 6) = 6.

Prime numbers are (2, 3, 5) = 3 (These are favourable outcomes).

Probability $\mathbf{P(E)} = \frac{3}{6} = \frac{1}{2}$

**Q3. A box contains 3 blue, 2 white and 4 red balls. What is the probability of drawing a white ball randomly from the box?**

**Answer-** Total number of balls = 3 + 2 + 4 = 9 (Total outcomes).

Number of white balls = 2 (Favourable outcomes).

According to the formula, Probability P (White)= $\frac{Favourable\ outcomes}{Total\ outcomes} = \frac{2}{9}$

**Q4. A card is drawn from a well-shuffled deck of 52 cards. Find the probability of getting an 'Ace'.**

**Answer-** Total number of cards in the deck = 52.

Total number of aces = 4 (Favourable outcomes).

Probability P(E) $= \frac{4}{52} = \frac{1}{13}$

**Q5. Two coins are tossed simultaneously. What will be the probability of getting at least one 'Head'?**

**Answer-** Total possible outcomes: {HH, HT, TH, TT} = 4.

Favourable outcomes with at least one head (HH, HT, TH) = 3.

Therefore, Probability $P(E) = \frac{3}{4}$.