NIOS PYQ's SOLUTIONS

PREVIOUS YEARS' QUESTIONS & ANSWERS

APRIL-2025

Your Path to Success

SECTION - A

Q1 - The distance of the point (4, –6) from the line $4x - 5y - 32 = 0$ is

(A) $\frac{3}{7}$

(B) $\frac{14}{41}$

(C) $\frac{7}{5}$

(D) $\frac{14}{\sqrt{41}}$

Answer - (D) $\frac{14}{\sqrt{41}}$

Q2 – If A is a square matrix such that $A^2 = A$, then $(I + A)^2 - 3A$ is equal to

(A) I

(B) $2A$

(C) $3I$

(D) A

Answer – (A) I

Q3 – If $A = \begin{bmatrix} cos\alpha & -sin\alpha \\ sin\alpha & cos\alpha \end{bmatrix}$ and $A + A' = I$, then the value of α is

(A) $\frac{\pi}{2}$

(B) $\frac{\pi}{3}$

(C) $\frac{\pi}{4}$

(D) $\frac{\pi}{6}$

Answer - (B) $\frac{\pi}{3}$

Q4 – The value of $\begin{vmatrix} 6 & 0 & -1 \\ 2 & 1 & 4 \\ 1 & 1 & 3 \end{vmatrix}$ is

(A) – 7

(B) 7

(C) 8

(D) 10

Answer - (A) – 7

Q5 – If $A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$, then A^{-1} is

(A) $\begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}$

(B) $\begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$

(C) $\begin{bmatrix} -3 & 5 \\ 1 & -2 \end{bmatrix}$

(D) $\begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix}$

Answer - (B) $\begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$

Q6 - The value of k, for which the matrix $\begin{bmatrix} k & 2 \\ 3 & 4 \end{bmatrix}$ is invertible, is

(A) $k = \frac{2}{3}$

(B) $k \neq \frac{2}{3}$

(C) $k \neq \frac{3}{2}$

(D) $k = \frac{3}{2}$

Answer - (C) $k \neq \frac{3}{2}$

Q7 – Given set A = {1, 2, 3}. A reflexive relation in set A is

(A) R = {(1, 2), (1, 3)}

(B) R = {(1, 1), (2, 2), (3, 3)}

(C) R = {(1, 1), (2, 2), (3, 1), (1, 3)}

(D) R = {(3, 1), (2, 1), (1, 1)}

Answer - (B) R = {(1, 1), (2, 2), (3, 3)}

Q8 – If $f(x) = x^3$ and $g(x) = cos3x$, then $f \circ g(x)$ is

(A) $x^3.cos3x$

(B) $cos3x^3$

(C) $cos^3 3x$

(D) $3cosx^3$

Answer - (C) $cos^3 3x$

Q9 - $\lim_{x \to 0} \frac{\sqrt{\frac{1}{2}(1 - cos2x)}}{x}$ is equal to

(A) 1

(B) – 1

(C) 0

(D) None of these

Answer - (D) None of these

Q10 – If $y = cotx°$, then $\frac{dy}{dx}$ is

(A) $cosecx°$

(B) $cosecx°.cotx°$

(C) $-1°cosec^2x°$

(D) $-1°cosecx°.cotx°$

Answer - (C) $-1°cosec^2x°$

Q11 – If $y = e^{-3logx}$, then $\frac{dy}{dx}$ is

(A) $-\frac{3}{x}$

(B) $-\frac{3}{x^2}$

(C) $-\frac{3}{x^4}$

(D) $-3x$

Answer - (C) $-\frac{3}{x^4}$

Q12 - The local minimum value of $y = x^3 - 3x + 2$ in the interval [0, 2] is

(A) 6

(B) 4

(C) 2

(D) 0

Answer - (D) 0

Q13 – If $\int 2^x dx = f(x) + C$, then $f(x)$ is

(A) 2^x

(B) $2xlog_e2$

(C) $\frac{2^x}{log_e2}$

(D) $\frac{2^{x+1}}{x+1}$

Answer - (C) $\frac{2^x}{log_e2}$

Q14 - The integrating factor of the differential equation $(x^2 - 1)\frac{dy}{dx} + 2xy = \frac{2}{x^2-1}$ is

(A) $2x$

(B) $x^2 - 1$

(C) $x - 1$

(D) $\frac{1}{x^2-1}$

Answer – (B) $x^2 - 1$

Q15 - The value of p, for which $p(\hat{\imath} + \hat{\jmath} + \hat{k})$ is a unit vector, is

(A) $\frac{1}{\sqrt{3}}$

(B) $-\frac{1}{\sqrt{3}}$

(C) $\pm\frac{1}{\sqrt{3}}$

(D) $\pm\sqrt{3}$

Answer - (C) $\pm\frac{1}{\sqrt{3}}$

Q16 - If for non-zero vectors $\vec{a}$ and $\vec{b}$, $\vec{a} \times \vec{b}$ is a unit vector and $|\vec{a}| = |\vec{b}| = \sqrt{2}$, then the angle between the vectors $\vec{a}$ and $\vec{b}$ is

(A) $\frac{\pi}{2}$

(B) $\frac{\pi}{3}$

(C) $\frac{\pi}{6}$

(D) $-\frac{\pi}{2}$

Answer – (C) $\frac{\pi}{6}$

Q17 – The value of $(\hat{\imath} \times \hat{\jmath}).\hat{k} + \hat{\imath}.\hat{\jmath}$ is

(A) 0

(B) 1

(C) 2

(D) -1

Answer - (B) 1

Q18 - The distance of the plane $3x - 4y + 12z = 3$ from the origin is

(A) $\frac{3}{13}$ *unit*

(B) $\frac{19}{13}$ *units*

(C) 3 *units*

(D) 1 *unit*

Answer - (A) $\frac{3}{13}$ *unit*

Q19 – The intercept cut by the plane $2x - y + 2z + 7 = 0$ on the x-axis is

(A) 2

(B) $\frac{7}{2}$

(C) $-\frac{7}{2}$

(D) -2

Answer - (C) $-\frac{7}{2}$

Q20 - Of all the points of the feasible region for maximum or minimum of the objective function, the point lies

(A) inside the feasible region

(B) at the boundary line of the feasible region

(C) at the corner point of the boundary of the feasible region

(D) None of the above

Answer - (C) at the corner point of the boundary of the feasible region

Q21 - Match Column - I statement with the correct option of Column – II.

Column-I	Column-II
For the ellipse $\frac{x^2}{36}+\frac{y^2}{16}=1$	
(a) the coordinates of foci are	**(P)** $\frac{\sqrt{5}}{3}$
(b) the eccentricity is	**(Q)** $(0, \pm 2\sqrt{5})$
	(R) $(\pm 2\sqrt{5}, 0)$
	(S) $3/\sqrt{5}$

Answer –

(a) the coordinates of foci are → (R) $(\pm 2\sqrt{5}, 0)$

(b) **the eccentricity is**→ (P) $\frac{\sqrt{5}}{3}$

Q22 - Fill in the blanks :

(a) If $f(x)=\frac{x-1}{|x-1|}, x\ (\neq 1) \in R$**, then the range of** f **is ____.**

(b) If $f: R \to R$ **is defined as** $f(x)=2x^3-1$**, then** f^{-1} **is ____, if** f^{-1} **exists.**

Answer – (a) $\{-1, 1\}$

(b) $f^{-1}(x) = \left(\frac{x+1}{2}\right)^{\frac{1}{3}}$

Q23 - Write True for correct statements and False for incorrect statements :

(a) If $y = 500e^{7x} + 600e^{-7x}$, then $\frac{d^2y}{dx^2} = 49y$.

(b) $\int_2^3 3^x dx = 18$

Answer – (a) True

(b) False

Q24 - Answer the following questions :

(a) If $f(x) = \frac{1}{4x^2+2x+1}$, then find its maximum value.

(b) Using differentials, find the approximate value of $\sqrt{49.5}$.

Answer – (a) $\frac{4}{3}$

(b) 7.036

Q25 - Fill in the blanks :

(a) The equation of the line which passes through the point (1, –2) and cuts off equal intercepts on the axes is ____.

Answer – $x + y + 1 = 0$

(b) The radius of the circle $x^2 + y^2 - 8x + 10y - 12 = 0$ is ____.

Answer – $\sqrt{53}$

(c) The length of the latus rectum of the parabola $y^2 = 24x$ is ____.

Answer – 24.

(d) The eccentricity of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, which passes through the points $(3, 0)$ and $(3\sqrt{2}, 2)$, is ____.

Answer – $\frac{\sqrt{13}}{3}$.

Q26 - Fill in the blanks :

(a) If A is a square matrix of order 2, then $|kA|$ is equal to ____.

Answer – $k^2|A|$

(b) If $F(x) = \begin{bmatrix} cosx & sinx \\ -sinx & cosx \end{bmatrix}$, then $F(x)F(y)$ is equal to F (____).

Answer – $(x + y)$

(c) If $x \in N$ and $\begin{bmatrix} x+3 & -2 \\ -3x & 2x \end{bmatrix} = 8$, then the value of x is ____.

Answer – 2

(d) If $A = \begin{bmatrix} 5 & 6 & -3 \\ -4 & 3 & 2 \\ -4 & -7 & 3 \end{bmatrix}$, then the cofactor of element (-7) is ____.

Answer – 2

Q27 - Write True for correct statements and False for incorrect statements :

(a) Rolle's theorem is applicable for the function $f(x) = x$ in the interval $[-1, 1]$.

Answer – False

(b) $\frac{d}{dx}(x^x) = x^x(1 + logx)$

Answer – True

(c) $\int_{-\frac{1}{2}}^{\frac{1}{2}} cos\, xlog\left(\frac{1+x}{1-x}\right) dx = 1$

Answer – True

(d) If p and q are the degree and order of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 + 3\frac{dy}{dx} + \frac{d^3y}{dx^3} = 4$ respectively, then $(2p - 3q)$ is (-2).

Answer – False

Q28 - Answer the following questions :

(a) What is the general solution of the differential equation $log\left(\frac{dy}{dx}\right) = 2x + y$?

Answer – $-e^{-y} = \frac{e^{2x}}{2} + C$

(b) Find the equation of the normal to the curve $ay^2 = x^3$ at the point (am^2, am^3).

Answer – $y - am^3 = -\frac{2}{3m}(x - am^2)$

(c) Find the area of the region bounded by the curve $y = x^2$ and the line $y = 4$.

Answer – $\frac{32}{3}$

(d) Find the derivative of $sinx\ w.r.t.\ logx$.

Answer – $xcos\ x$

Q29 - Let $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}\ and\ \vec{b} = \hat{\imath} + \hat{\jmath}$ be the two vectors. Then—

(a) find $\vec{a}.\vec{b}$;

(b) find the unit vector perpendicular to both the vector $\vec{a}\ and\ \vec{b}$;

(c) find the area of the parallelogram having $\vec{a}\ and\ \vec{b}$ as diagonals;

(d) for the given vectors $\vec{a}\ and\ \vec{b}$, verify $(\vec{a} \times \vec{b})^2 = \vec{a}^2\vec{b}^2 - (\vec{a}.\vec{b})^2$;

(e) find $|2\vec{b} \times \vec{a}|$;

(f) for the given vectors, check if $\vec{a} \times \vec{b} = \vec{b} \times \vec{a}$.

Answer – (a) $\vec{a}.\vec{b} = 2$

(b) $|\vec{a} \times \vec{b}| = -\hat{k}$

(c) $Area = \frac{1}{2}|\vec{a} \times \vec{b}| = \frac{\sqrt{2}}{2}$

(d) Verified $\left[Idenity :\ (\vec{a} \times \vec{b})^2 = |\vec{a}||\vec{b}| - (\vec{a}.\vec{b})^2 = \vec{a}^2\vec{b}^2 - (\vec{a}.\vec{b})^2\right]$

(e) $\left|2\vec{b} \times \vec{a}\right| = 2\sqrt{2}$

(f) $\vec{a} \times \vec{b} \neq \vec{b} \times \vec{a}$

SECTION - B

Q30 - Find the equation of the line passing through the intersection of the lines $x + y = 5$ and $2x - y - 7 = 0$, and parallel to x-axis.

Answer –

Find intersection of the lines:

$$x + y = 5$$

$$2x - y - 7 = 0$$

From first:

$$y = 5 - x$$

Substitute in second:

$$2x - (5 - x) - 7 = 0$$

$$2x - 5 + x - 7 = 0$$

$$3x - 12 = 0$$

$$x = 4$$

$$y = 5 - 4 = 1$$

Intersection point: $(4, 1)$

A line parallel to x-axis has equation:

$$y = \text{constant}$$

Hence required line:

$$\boxed{y = 1}$$

OR

Find the distance between the lines $2x + 3y = 4$ and $4x + 6y = 20$.

Answer –

Given lines:

$$2x + 3y - 4 = 0$$

$$4x + 6y - 20 = 0$$

First make coefficients of x and y same.

Divide second equation by 2:

$$2x + 3y - 10 = 0$$

Now the lines are:

$$2x + 3y - 4 = 0$$

$$2x + 3y - 10 = 0$$

These are parallel lines.

Distance between parallel lines:

$$d = \frac{|c_1 - c_2|}{\sqrt{a^2 + b^2}}$$

Here,

$$c_1 = -4, \quad c_2 = -10$$

$$d = \frac{|-4 - (-10)|}{\sqrt{2^2 + 3^2}}$$

$$= \frac{6}{\sqrt{13}}$$

Q31 - Find the equation of the hyperbola whose vertices are $(\pm 2, 0)$ and foci are $(\pm 3, 0)$.

Answer –

Vertices are $(\pm 2, 0)$.

So the hyperbola is centered at the origin and has transverse axis along the x-axis.

Standard form:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Since vertices are $(\pm a, 0)$,

$$a = 2 \Rightarrow a^2 = 4$$

Given focus $(3, 0)$, so

$$c = 3 \Rightarrow c^2 = 9$$

For hyperbola:

$$c^2 = a^2 + b^2$$

$$9 = 4 + b^2$$

$$b^2 = 5$$

Final Equation

$$\boxed{\frac{x^2}{4} - \frac{y^2}{5} = 1}$$

Q32 - For any matrix A of order 3×3, prove that $(A') = A$.

Answer –

Let

$$A = [a_{ij}] = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

The transpose of A is

$$A' = A^T = \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{bmatrix}$$

Now take transpose again:

$$(A')' = (A^T)^T$$

$$= \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$= A$$

Hence,

$$\boxed{(A')' = A}$$

Proved.

Q33 – Show that $\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix} = (a-b)(b-c)(c-a)$.

Answer –

Given

$$\Delta = \begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$$

Apply row operations:

$$R_2 \to R_2 - R_1, \qquad R_3 \to R_3 - R_1$$

$$= \begin{vmatrix} 1 & a & bc \\ 0 & b-a & ca-bc \\ 0 & c-a & ab-bc \end{vmatrix}$$

Now expand along first column:

$$\Delta = \begin{vmatrix} b-a & ca-bc \\ c-a & ab-bc \end{vmatrix}$$

Factor:

$$ca - bc = c(a-b), \qquad ab - bc = b(a-c)$$

So,

$$\Delta = \begin{vmatrix} b-a & c(a-b) \\ c-a & b(a-c) \end{vmatrix}$$

Take common factors:

$$b - a = -(a-b), \qquad c - a = -(a-c)$$

$$\Delta = (a-b)(b-c)(c-a)$$

Q34 – If $A = \{1,2,3\}$ and relation $R = \{(2,3)\}$ in A, check whether the relation R is reflexive, symmetric or transitive.

Answer –

Given

$$A = \{1, 2, 3\}$$

$$R = \{(2, 3)\}$$

Reflexive

A relation on A is reflexive if

$$(1,1),(2,2),(3,3) \in R$$

Since none of these are in R,

$$\boxed{\text{Not Reflexive}}$$

Symmetric

A relation is symmetric if

$$(2,3) \in R \Rightarrow (3,2) \in R$$

But $(3,2) \notin R$.

$$\boxed{\text{Not Symmetric}}$$

Transitive

A relation is transitive if

$$(a,b) \in R \text{ and } (b,c) \in R \Rightarrow (a,c) \in R$$

Here there is only one pair $(2,3)$, and no pair starting with 3.

So the condition is not violated.

$$\boxed{\text{Transitive}}$$

Q35 – Express the function $tan^{-1}\left[\frac{\sqrt{1+x^2}-1}{x}\right], x \neq 0$ in the simplest form.

Answer –

Given

$$\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right), \quad x \neq 0$$

Step 1: Multiply numerator and denominator by conjugate

$$\frac{\sqrt{1+x^2}-1}{x} = \frac{(\sqrt{1+x^2}-1)(\sqrt{1+x^2}+1)}{x(\sqrt{1+x^2}+1)}$$

$$= \frac{1+x^2-1}{x(\sqrt{1+x^2}+1)}$$

$$= \frac{x^2}{x(\sqrt{1+x^2}+1)}$$
$$= \frac{x}{\sqrt{1+x^2}+1}$$

So expression becomes

$$\tan^{-1}\left(\frac{x}{\sqrt{1+x^2}+1}\right)$$

Step 2: Use identity

$$\tan\frac{\theta}{2} = \frac{\sin\theta}{1+\cos\theta}$$

Let

$$x = \tan\theta$$

Then

$$\sqrt{1+x^2} = \sec\theta$$

So,

$$\frac{x}{\sqrt{1+x^2}+1} = \frac{\tan\theta}{\sec\theta+1} = \frac{\sin\theta}{1+\cos\theta} = \tan\frac{\theta}{2}$$

Thus,

$$\tan^{-1}\left(\frac{x}{\sqrt{1+x^2}+1}\right) = \frac{\theta}{2}$$

Since $\theta = \tan^{-1} x$,

$$\boxed{\frac{1}{2}\tan^{-1} x}$$

OR

Show that the function $f: R \to R$ defined by $f(x) = \begin{cases} 1, & if\ x > 0 \\ 0, & if\ x = 0 \\ -1, & if\ x < 0 \end{cases}$ is neither one-one nor onto.

Answer – Given

$$f : \mathbb{R} \to \mathbb{R}$$

$$f(x) = \begin{cases} 1, & x > 0 \\ 0, & x = 0 \\ -1, & x < 0 \end{cases}$$

Not One–One

For a function to be one–one,

$$f(x_1) = f(x_2) \Rightarrow x_1 = x_2$$

But,

$$f(1) = 1, \quad f(2) = 1$$

and $1 \neq 2$.

Hence,

$$\boxed{\text{f is not one–one}}$$

Not Onto

Range of f is

$$\{-1, 0, 1\}$$

But codomain is $\mathbb{R}$.

Since values like $2, \frac{1}{2}, -3$ etc. are not attained,

$$\boxed{\text{f is not onto}}$$

Final Conclusion

$$\boxed{\text{f is neither one–one nor onto}}$$

Q36 – Show that $y = log(1 + x) - \frac{2x}{2+x}, (x > -1)$ is an increasing function of x throughout its domain.

Answer –

Given

$$y = \log(1 + x) - \frac{2x}{2 + x}, \qquad (x > -1)$$

Step 1: Differentiate

$$\frac{dy}{dx} = \frac{1}{1 + x} - \frac{d}{dx}\left(\frac{2x}{2 + x}\right)$$

Using quotient rule:

$$\frac{d}{dx}\left(\frac{2x}{2 + x}\right) = \frac{2(2 + x) - 2x(1)}{(2 + x)^2}$$

Step 2: Simplify

Take common denominator:

$$\frac{dy}{dx} = \frac{(2+x)^2 - 4(1+x)}{(1+x)(2+x)^2}$$

Expand numerator:

$$(2+x)^2 = x^2 + 4x + 4$$

$$x^2 + 4x + 4 - 4 - 4x = x^2$$

Thus,

$$\frac{dy}{dx} = \frac{x^2}{(1+x)(2+x)^2}$$

Step 3: Sign of derivative

For $x > -1$:

$$(1+x) > 0, \quad (2+x)^2 > 0$$

$$x^2 \geq 0$$

Hence,

$$\frac{dy}{dx} \geq 0$$

and > 0 for $x \neq 0$.

Final Conclusion

$$\boxed{y \text{ is an increasing function for } x > -1}$$

OR

Prove that the curves $y^2 = 4ax$ and $xy = k^2$ cut at right angles, if $k^4 = 32a^4$.

Answer –

Given curves

$$y^2 = 4ax$$

$$xy = k^2$$

We have to show they cut at right angles if

$$k^2 = 32a^4$$

Step 1: Slope of first curve

Differentiate $y^2 = 4ax$:

$$2y\frac{dy}{dx} = 4a$$

$$\frac{dy}{dx} = \frac{2a}{y}$$

Step 2: Slope of second curve

Differentiate $xy = k^2$:

$$x\frac{dy}{dx} + y = 0$$

$$\frac{dy}{dx} = -\frac{y}{x}$$

Step 3: Condition for perpendicularity

If slopes are m_1 and m_2, then

$$m_1 m_2 = -1$$

So,

$$\frac{2a}{y}\left(-\frac{y}{x}\right) = -1$$

$$-\frac{2a}{x} = -1$$

$$x = 2a$$

Step 4: Find intersection values

From parabola:

$$y^2 = 4a(2a)$$

$$y^2 = 8a^2$$

$$y = \pm 2\sqrt{2}\,a$$

Step 5: Use in second curve

$$xy = k^2$$

$$(2a)(\pm 2\sqrt{2}\,a) = k^2$$

$$k^2 = \pm 4\sqrt{2}\,a^2$$

Final Result

$$\boxed{k^4 = 32a^4}$$

Hence the curves cut at right angles if $k^4 = 32a^4$.

Q37 - If $|\vec{a}| = 2, |\vec{b}| = 3$ and $\vec{a}.\vec{b} = 4$, then find $|\vec{b} - \vec{a}|$.

Answer -

Given

$$|\vec{a}| = 2, \quad |\vec{b}| = 3, \quad \vec{a} \cdot \vec{b} = 4$$

We use identity:

$$|\vec{b} - \vec{a}|^2 = |\vec{b}|^2 + |\vec{a}|^2 - 2(\vec{a} \cdot \vec{b})$$

Substitute values:

$$= 3^2 + 2^2 - 2(4)$$

$$= 9 + 4 - 8$$

$$= 5$$

Therefore,

$$|\vec{b} - \vec{a}| = \sqrt{5}$$

OR

Find the points on the line $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2}$ at a distance $3\sqrt{2}$ from the point $(1, 2, 3)$.

Answer –

Given line:

$$\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2}$$

Let the common value be t.

$$x = -2 + 3t$$

$$y = -1 + 2t$$

$$z = 3 + 2t$$

So any point on line is

$$P(-2 + 3t,\ -1 + 2t,\ 3 + 2t)$$

Given point $A(1, 2, 3)$.

Distance condition:

$$PA = 3\sqrt{2}$$

So,

$$(-2 + 3t - 1)^2 + (-1 + 2t - 2)^2 + (3 + 2t - 3)^2 = (3\sqrt{2})^2$$
$$(-3 + 3t)^2 + (-3 + 2t)^2 + (2t)^2 = 18$$

Expand:

$$9(t-1)^2 + (2t-3)^2 + 4t^2 = 18$$
$$9(t^2 - 2t + 1) + (4t^2 - 12t + 9) + 4t^2 = 18$$
$$9t^2 - 18t + 9 + 4t^2 - 12t + 9 + 4t^2 = 18$$
$$17t^2 - 30t + 18 = 18$$
$$17t^2 - 30t = 0$$
$$t(17t - 30) = 0$$
$$t = 0 \quad \text{or} \quad t = \frac{30}{17}$$

Points

For $t = 0$:

$$(-2, -1, 3)$$

For $t = \frac{30}{17}$:

$$\left(\frac{56}{17}, \frac{43}{17}, \frac{111}{17}\right)$$

Q38 - Write the following statements in the form 'if · · · then' :

(a) It never rains when it is cold.

(b) A quadrilateral is a parallelogram if its diagonals bisect each other.

Answer –

(a)

Original statement:

"It never rains when it is cold."

If–then form:

$$\boxed{\text{If it is cold, then it does not rain.}}$$

(b)

Original statement:

"A quadrilateral is a parallelogram if its diagonals bisect each other."

If–then form:

$$\boxed{\text{If the diagonals of a quadrilateral bisect each other, then it is a parallelogram.}}$$

Q39 - If the eccentricity of an ellipse is $\frac{5}{8}$ and the distance between its foci is 10, then find the length of the latus rectum of the ellipse.

Answer –

Given

$$e = \frac{5}{8}$$

Distance between foci:

$$2c = 10$$

$$c = 5$$

For ellipse:

$$e = \frac{c}{a}$$

$$\frac{5}{8} = \frac{5}{a}$$

$$a = 8$$

Now,

$$b^2 = a^2 - c^2$$

$$= 64 - 25$$

$$= 39$$

Length of latus rectum:

$$\boxed{\frac{2b^2}{a} = \frac{2(39)}{8} = \frac{78}{8} = \frac{39}{4}}$$

Q40 - Using matrix method, find the solution of the following system of linear equations :

$$2x - 3y + 5z = 16$$
$$3x + 2y - 4z = -4$$
$$x + y - 2z = -3$$

Answer –

Step 1: Write in matrix form

$$AX = B$$

$$\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 16 \\ -4 \\ -3 \end{bmatrix}$$

Step 2: Solve using elimination (equivalent to matrix method)

From third equation:

$$x = -3 - y + 2z$$

Substitute into first two equations.

In (1):

$$2(-3 - y + 2z) - 3y + 5z = 16$$

$$-6 - 2y + 4z - 3y + 5z = 16$$

$$-6 - 5y + 9z = 16$$

$$-5y + 9z = 22$$

$$5y - 9z = -22 \quad (i)$$

In (2):

$$3(-3 - y + 2z) + 2y - 4z = -4$$

$$-9 - 3y + 6z + 2y - 4z = -4$$

$$-9 - y + 2z = -4$$

$$-y + 2z = 5$$

$$y - 2z = -5 \quad (ii)$$

Step 3: Solve (i) and (ii)

From (ii):

$$y = -5 + 2z$$

Substitute in (i):

$$5(-5 + 2z) - 9z = -22$$

$$-25 + 10z - 9z = -22$$

$$-25 + z = -22$$

$$z = 3$$

$$y = -5 + 2(3) = 1$$

Step 4: Find x

$$x = -3 - y + 2z$$

$$= -3 - 1 + 6$$

$$= 2$$

Final Answer

$$\boxed{x = 2, \quad y = 1, \quad z = 3}$$

Q41 - Let N be the set of all natural numbers and let R be a relation on N × N defined by $(a, b)\ R(c, d) \Leftrightarrow ad = bc\ \forall (a, b), (c, d) \in N \times N$. Show that R is an equivalence relation on N × N.

Answer –

Given

N = set of natural numbers

Relation R on $N \times N$ defined by

$$(a, b)R(c, d) \iff ad = bc$$

We prove R is an equivalence relation.

1. Reflexive

For any $(a, b) \in N \times N$,

$$ad = bc \quad \text{when} \quad (c, d) = (a, b)$$

$$ab = ba$$

True.

Hence,

$$\boxed{\text{R is reflexive}}$$

2. Symmetric

If

$$(a, b)R(c, d)$$

then

$$ad = bc$$

This implies

$$bc = ad$$

So,

$$(c, d)R(a, b)$$

Hence,

$$\boxed{\text{R is symmetric}}$$

3. Transitive

If

$$(a, b)R(c, d) \Rightarrow ad = bc$$

and

$$(c, d)R(e, f) \Rightarrow cf = de$$

We need to prove

$$(a, b)R(e, f)$$

Multiply:

$$ad = bc \quad \Rightarrow \quad adf = bcf$$
$$cf = de \quad \Rightarrow \quad bcf = bde$$

Thus,

$$adf = bde$$

Cancel d (since natural numbers are non-zero):

$$af = be$$

Hence,

$$(a, b)R(e, f)$$

So,

$$\boxed{\text{R is transitive}}$$

Final Conclusion

Since R is reflexive, symmetric and transitive,

$$\boxed{\text{R is an equivalence relation on } N \times N}$$

Q42 – Prove that the curves $y^2 = 4x$ and $x^2 = 4y$ divide the area of the square bounded by $x = 0, x = 4, y = 4$ *and* $y = 0$ into three equal parts.

Answer –

Given curves:

$$y^2 = 4x \quad \text{and} \quad x^2 = 4y$$

Step 1: Find points of intersection

From $y^2 = 4x$,

$$x = \frac{y^2}{4}$$

Substitute in $x^2 = 4y$:

$$\left(\frac{y^2}{4}\right)^2 = 4y$$

$$\frac{y^4}{16} = 4y$$

$$y^4 = 64y$$

$$y(y^3 - 64) = 0$$

$$y = 0 \quad \text{or} \quad y = 4$$

Corresponding x:

$$(0, 0) \quad \text{and} \quad (4, 4)$$

Thus they divide the square bounded by $x = 0, y = 0, x = 4, y = 4$.

Area of square:

$$= 4 \times 4 = 16$$

Step 2: Area between curves

Using

$$x = 2\sqrt{y} \quad \text{and} \quad x = \frac{y^2}{4}$$

Area enclosed:

$$A = \int_0^4 \left(2\sqrt{y} - \frac{y^2}{4}\right) dy$$

$$= \left[\frac{4}{3}y^{3/2} - \frac{y^3}{12}\right]_0^4$$

$$= \frac{4}{3}(8) - \frac{64}{12}$$

$$= \frac{32}{3} - \frac{16}{3}$$

$$= \frac{16}{3}$$

Step 3: Remaining area

Total square area:

$$16$$

Remaining area:

$$16 - \frac{16}{3} = \frac{48 - 16}{3} = \frac{32}{3}$$

This area is divided into two equal parts by symmetry:

$$\frac{32}{3} \div 2 = \frac{16}{3}$$

Final Result

Each region has area

$$\boxed{\frac{16}{3}}$$

Hence the curves divide the square into three equal areas.

OR

Solve $sec^2y(1+x^2)dy + 2x\,tan\,ydx = 0$, given that $y = \frac{\pi}{4}$, when $x = 1$.

Answer –

Given differential equation:

$$\sec^2 y(1+x^2)\,dy + 2x\tan y\,dx = 0$$

Step 1: Rearrange

$$\sec^2 y(1+x^2)\,dy = -2x\tan y\,dx$$

Divide both sides by $(1+x^2)\tan y$:

$$\frac{\sec^2 y}{\tan y}\,dy = -\frac{2x}{1+x^2}\,dx$$

Step 2: Simplify left side

$$\frac{\sec^2 y}{\tan y} = \frac{1/\cos^2 y}{\sin y/\cos y} = \frac{1}{\sin y\cos y}$$

So,

$$\frac{dy}{\sin y\cos y} = -\frac{2x}{1+x^2}\,dx$$

Step 3: Integrate

Left side:

$$\int\frac{dy}{\sin y\cos y} = \int\frac{d(\tan y)}{\tan y} = \ln|\tan y|$$

Right side:

$$\int -\frac{2x}{1+x^2}\,dx = -\ln(1+x^2)$$

Step 4: Combine

$$\ln|\tan y| = -\ln(1+x^2) + C$$

$$\ln|\tan y| = \ln\frac{C}{1+x^2}$$

$$\tan y = \frac{C}{1+x^2}$$

Step 5: Use initial condition

Given $y = \frac{\pi}{4}$ when $x = 1$.

$$\tan\frac{\pi}{4} = 1$$

$$1 = \frac{C}{1+1}$$

$$1 = \frac{C}{2}$$

$$C = 2$$

Final Answer

$$\boxed{\tan y = \frac{2}{1+x^2}}$$

Q43 - Show that the four points A, B, C and D, whose position vectors are $4\hat{\imath} + 5\hat{\jmath} + \hat{k}, i - \hat{\jmath} - \hat{k}$, $3\hat{\imath} + 9\hat{\jmath} + 4\hat{k}$ *and* $-4\hat{\imath} + 4\hat{\jmath} + 4\hat{k}$, are coplanar.

Answer –

Position vectors:

$$\vec{A} = 4\hat{i} + 5\hat{j} + \hat{k}$$

$$\vec{B} = \hat{i} - \hat{j} - \hat{k}$$

$$\vec{C} = 3\hat{i} + 9\hat{j} + 4\hat{k}$$

$$\vec{D} = -4\hat{i} + 4\hat{j} + 4\hat{k}$$

Step 1: Form three vectors

$$\vec{AB} = \vec{B} - \vec{A} = (-3, -6, -2)$$

$$\vec{AC} = \vec{C} - \vec{A} = (-1, 4, 3)$$

$$\vec{AD} = \vec{D} - \vec{A} = (-8, -1, 3)$$

Step 2: Compute scalar triple product

$$[\vec{AB}\ \vec{AC}\ \vec{AD}] = \begin{vmatrix} -3 & -6 & -2 \\ -1 & 4 & 3 \\ -8 & -1 & 3 \end{vmatrix}$$

Expanding:

$$= -3\begin{vmatrix} 4 & 3 \\ -1 & 3 \end{vmatrix} + 6\begin{vmatrix} -1 & 3 \\ -8 & 3 \end{vmatrix} - 2\begin{vmatrix} -1 & 4 \\ -8 & -1 \end{vmatrix}$$

$$= -3(12+3) + 6(-3+24) - 2(1+32)$$

$$= -45 + 126 - 66$$

$$= 0$$

Since scalar triple product is zero,

$$\boxed{\text{The four points are coplanar.}}$$

OR

Reduce the equation of the plane $4x - 5y + 6z = 60 = 0$ to the

(a) intercept form and find its intercepts on the axes and

(b) normal form and find the length of the perpendicular from the origin to the plane.

Answer –

Given plane:

$$4x - 5y + 6z - 60 = 0$$

$$4x - 5y + 6z = 60$$

(a) Intercept form

Divide by 60:

$$\frac{4x}{60} - \frac{5y}{60} + \frac{6z}{60} = 1$$

$$\frac{x}{15} - \frac{y}{12} + \frac{z}{10} = 1$$

So intercept form:

$$\boxed{\frac{x}{15} - \frac{y}{12} + \frac{z}{10} = 1}$$

Intercepts:

$$(15, 0, 0), \quad (0, -12, 0), \quad (0, 0, 10)$$

(b) Normal form

Given plane:

$$4x - 5y + 6z - 60 = 0$$

Magnitude of normal vector:

$$\sqrt{4^2 + (-5)^2 + 6^2} = \sqrt{16 + 25 + 36} = \sqrt{77}$$

Divide entire equation by $\sqrt{77}$:

$$\frac{4}{\sqrt{77}}x - \frac{5}{\sqrt{77}}y + \frac{6}{\sqrt{77}}z = \frac{60}{\sqrt{77}}$$

Normal form:

$$\boxed{x\frac{4}{\sqrt{77}} - y\frac{5}{\sqrt{77}} + z\frac{6}{\sqrt{77}} = \frac{60}{\sqrt{77}}}$$

Q44 - Verify Lagrange's mean value theorem for the function $f(x) = (x-1)(x-2)(x-3)$ in the interval $[0, 4]$.

Answer –

Given

$$f(x) = (x-1)(x-2)(x-3)$$

on interval $[0, 4]$.

Step 1: Check conditions

$f(x)$ is a polynomial, hence continuous on $[0, 4]$ and differentiable on $(0, 4)$.

So Lagrange's Mean Value Theorem is applicable.

Step 2: Compute $f(0)$ and $f(4)$

$$f(0) = (-1)(-2)(-3) = -6$$

$$f(4) = (3)(2)(1) = 6$$

$$\frac{f(4) - f(0)}{4 - 0} = \frac{6 - (-6)}{4} = \frac{12}{4} = 3$$

Step 3: Find $f'(x)$

Expand first:

$$f(x) = x^3 - 6x^2 + 11x - 6$$

$$f'(x) = 3x^2 - 12x + 11$$

Set

$$f'(c) = 3$$

$$3x^2 - 12x + 11 = 3$$

$$3x^2 - 12x + 8 = 0$$

Step 4: Solve

$$x = \frac{12 \pm \sqrt{144 - 96}}{6}$$

$$= \frac{12 \pm \sqrt{48}}{6}$$

$$= \frac{12 \pm 4\sqrt{3}}{6}$$

$$= 2 \pm \frac{2\sqrt{3}}{3}$$

Both values lie in $(0, 4)$.

Final Answer

$$\boxed{c = 2 \pm \frac{2\sqrt{3}}{3}}$$

Hence Lagrange's Mean Value Theorem is verified.

OR

Verify Rolle's theorem for the function $f(x) = x(x-1)(x-2)$ in the interval $[0, 2]$.

Answer -

Given

$$f(x) = x(x-1)(x-2)$$

in the interval $[0, 2]$.

Step 1: Check conditions of Rolle's Theorem

Since $f(x)$ is a polynomial, it is continuous on $[0, 2]$ and differentiable on $(0, 2)$.

Now,

$$f(0) = 0$$

$$f(2) = 2(1)(0) = 0$$

Thus,

$$f(0) = f(2)$$

Hence all conditions of Rolle's Theorem are satisfied.

Step 2: Find $f'(x)$

First expand:

$$f(x) = x^3 - 3x^2 + 2x$$

$$f'(x) = 3x^2 - 6x + 2$$

Set

$$f'(c) = 0$$

$$3x^2 - 6x + 2 = 0$$

Step 3: Solve

$$x = \frac{6 \pm \sqrt{36 - 24}}{6}$$

$$= \frac{6 \pm \sqrt{12}}{6}$$

$$= \frac{6 \pm 2\sqrt{3}}{6}$$

$$= 1 \pm \frac{\sqrt{3}}{3}$$

Both values lie in $(0, 2)$.

Final Answer

$$\boxed{c = 1 \pm \frac{\sqrt{3}}{3}}$$

Hence Rolle's Theorem is verified.

Q45 - Minimize and maximize $Z = 5x + 2y$ subject to the following constraints :

$$x - 2y \leq 2, \quad 3x + 2y \leq 12, \quad -3x + 2y \leq 3$$

$$x \geq 0, \quad y \geq 0$$

Answer –

Objective Function

$$Z = 5x + 2y$$

Subject to Constraints

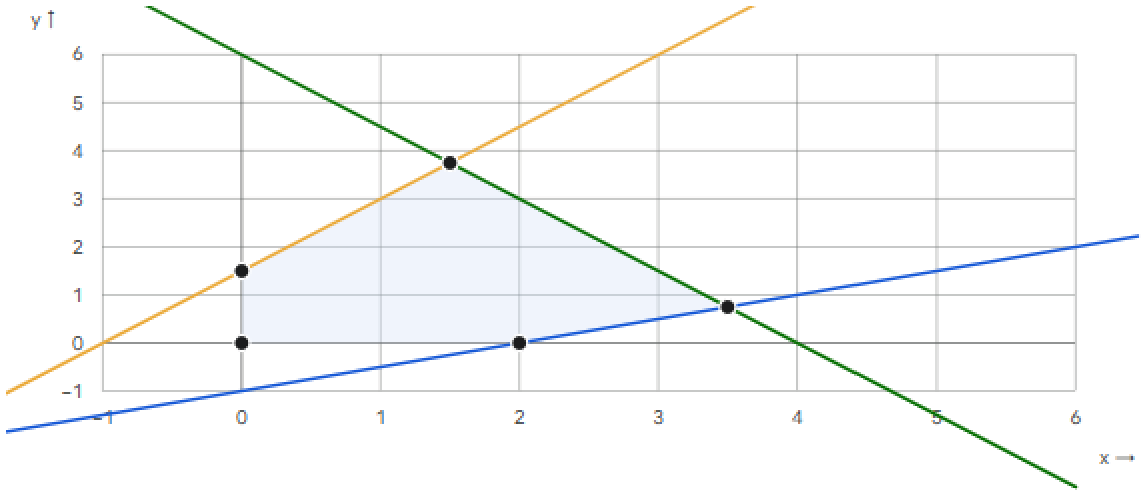
$$x - 2y \leq 2$$

$$3x + 2y \leq 12$$

$$-3x + 2y \leq 3$$

$$x \geq 0,\ y \geq 0$$

Graphical Solution of LPP



Graphical Solution Steps (Plotting the LPP):

- **Line 1 ($x - 2y = 2$):** Passes through $(2, 0)$ and $(0, -1)$. Test $(0, 0) \Rightarrow 0 \leq 2$ (True, shade towards origin).
- **Line 2 ($3x + 2y = 12$):** Passes through $(4, 0)$ and $(0, 6)$. Test $(0, 0) \Rightarrow 0 \leq 12$ (True, shade towards origin).
- **Line 3 ($-3x + 2y = 3$):** Passes through $(-1, 0)$ and $(0, 1.5)$. Test $(0, 0) \Rightarrow 0 \leq 3$ (True, shade towards origin).
- **Feasible Region:** The bounded polygon in the first quadrant (since $x, y \geq 0$) formed by the intersection of these active constraint lines.

Corner Points (from graph)	Evaluate Z
$(0, 0)$	$Z(0, 0) = 5(0) + 2(0) = \mathbf{0}$
$(2, 0)$	$Z(2, 0) = 5(2) + 2(0) = 10$
$(3.5, 0.75)$	$Z(3.5, 0.75) = 5(3.5) + 2(0.75) = \mathbf{19}$
$(1.5, 3.75)$	$Z(1.5, 3.75) = 5(1.5) + 2(3.75) = 15$
$(0, 1.5)$	$Z(0, 1.5) = 5(0) + 2(1.5) = 3$

Final Answer

$Z_{min} = 0$ at $(0, 0)$

$Z_{max} = 19$ at $(3.5, 0.75)$

Hence, minimum value of Z is 0 and maximum value is 19.

OR

A farmer has a supply of two chemical fertilizers A and B. Fertilizer of type A contains 10% nitrogen and 5% phosphoric acid. Fertilizer of type B contains 6% nitrogen and 10% phosphoric acid. After testing the soil conditions of the field, it is found that at least 14 kg of nitrogen and 14 kg of phosphoric acid are required for producing a good crop. Fertilizer of type A costs ₹5 per kg and type B costs ₹3 per kg. How many kg of each type of fertilizer should be used to meet the requirement at the minimum possible cost? Formulate the situation as an LPP. Also, obtain the feasible region corresponding to the constraints.

Answer –

Let

$$x = \text{kg of fertilizer A}$$

$$y = \text{kg of fertilizer B}$$

Formulation of LPP

Objective Function (Minimize Cost)

$$Z = 5x + 3y$$

Constraints

Nitrogen requirement:

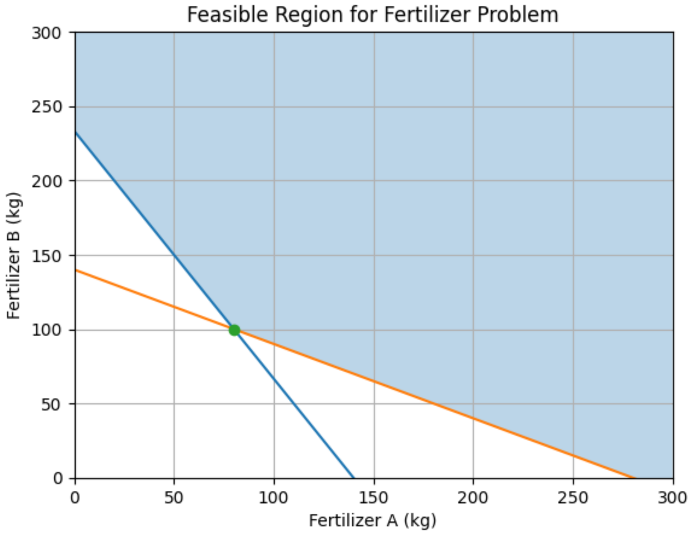
$$0.10x + 0.06y \geq 14$$

Phosphoric acid requirement:

$$0.05x + 0.10y \geq 14$$

Non-negativity:

$$x \geq 0, \quad y \geq 0$$



Corner Point (from graph)

Solve

$$0.10x + 0.06y = 14$$

$$0.05x + 0.10y = 14$$

Multiplying first by 5 and second by 10:

$$0.50x + 0.30y = 70$$

$$0.50x + 1.00y = 140$$

Subtracting:

$$0.70y = 70$$

$$y = 100$$

Substitute:

$$0.05x + 0.10(100) = 14$$

$$0.05x + 10 = 14$$

$$0.05x = 4$$

$$x = 80$$

Minimum Cost

$$Z = 5(80) + 3(100)$$

$$= 400 + 300$$

$$= 700$$

Final Conclusion

$$\boxed{80 \text{ kg of A and } 100 \text{ kg of B}}$$

$$\boxed{\text{Minimum Cost } = ₹700}$$

The feasible region is unbounded, but the minimum cost occurs at the intersection of the two nutrient constraint lines.

NIOS PYQ's SOLUTIONS

PREVIOUS YEARS' QUESTIONS & ANSWERS

OCTOBER-2025

Your Path to Success

SECTION - A

Q1 - The ratio in which the point P whose ordinate is –3 divides the join of A (6, 5) and B (-1, 4) is

(A) 7:8 externally

(B) 7:8 internally

(C) 8:7 externally

(D) 8:7 internally

Answer - (C) 8:7 externally

Q2 – Distance between the lines $3x + 4y = 5$ and $3x + 4y + 5 = 0$

(A) 10 units

(B) 2 units

(C) 1 unit

(D) 4 units

Answer – (B) 2 units

Q3 – If (3, 4) is one extremity of a diameter of the circle $x^2 + y^2 + 2x - 4y - 15 = 0$, then the coordinates of the other extremity are

(A) $(5, 0)$

(B) $(0, 5)$

(C) $(-5, 0)$

(D) $(0, -5)$

Answer - (C) $(-5, 0)$

Q4 – The foci of the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$ is

(A) $(\pm 7, 0)$

(B) $(0, \pm 7)$

(C) $(0, \pm 5)$

(D) $(\pm 5, 0)$

Answer - (D) $(\pm 5,0)$

Q5 – If the length of the major axis of an ellipse is 3 times the length of its minor axis, then the eccentricity of the ellipse is

(A) $\frac{2\sqrt{2}}{3}$

(B) $\frac{3}{2\sqrt{2}}$

(C) $\frac{\sqrt{2}}{3}$

(D) $\frac{1}{2\sqrt{3}}$

Answer - (A) $\frac{2\sqrt{2}}{3}$

Q6 - If $A = \begin{bmatrix} 2 & -1 & 3 \\ -4 & 5 & 1 \end{bmatrix}$ **and** $B = \begin{bmatrix} 2 & 3 \\ 4 & -2 \\ 1 & 5 \end{bmatrix}$ **are two matrices, then**

(A) Only AB is defined

(B) Only BA is defined

(C) AB and BA both are not defined

(D) AB and BA both are defined

Answer - (D) AB and BA both are defined

Q7 – Let Z **be the set of all integers and** R **be the relation defined in** Z **such that** xRy **if** $|x - y| \leq 2$. **Then,** R **is a/an**

(A) Reflexive and transitive relation

(B) Reflexive and symmetric relation

(C) Symmetric and transitive relation

(D) Equivalence relation

Answer - (B) Reflexive and symmetric relation

Q8 – If $f: R \to R$ *and* $g: R \to R$ **are defined by** $f(x) = 8x^3$ *and* $g(x) = \sqrt[3]{x}$, **then** $(f \circ g)(x)$ **is**

(A) $8x$

(B) $2x$

(C) $2\sqrt[3]{x}$

(D) $8\sqrt[3]{x}$

Answer - (A) $8x$

Q9 - If A and B are two matrices such that $AB = O$ then

(A) It is not necessary that $A = O$ or $B = O$

(B) $A = O$ or $B = O$

(C) $A = O$ and $B = O$

(D) All the above are wrong

Answer - (A) It is not necessary that $A = O$ or $B = O$

Q10 – Let $A = \begin{bmatrix} 1 & w^2 & w^2 \\ w^2 & 1 & w \\ w^2 & w & 1 \end{bmatrix}$, where w is the cube root of 1. Then $det(A)$ is equal to

(A) $-w$

(B) $-w^2$

(C) -1

(D) $-3w$

Answer - (D) $-3w$

Q11 – P is a point on the join of the points $Q(2, 1, 2)$ and $R(5, 2, 5)$. If x-coordinate of P is 4, then its z-coordinate is

(A) 2

(B) 1

(C) -1

(D) 4

Answer - (D) 4

Q12 - The contrapositive of the statement "If you are born in India, then you are a citizen of India." is

(A) If you are a citizen of India, then you are born in India.

(B) If you are not a citizen of India, then you are not born in India.

(C) You are born in India if and only if you are a citizen of India.

(D) you are born in India or you are a citizen of India.

Answer - (B) If you are not a citizen of India, then you are not born in India.

Q13 – The domain of the function $f(x) = cos^{-1}(x^2 - 4)$ is

(A) $[-\sqrt{5}, -\sqrt{3}]$

(B) $[\sqrt{3}, \sqrt{5}]$

(C) $[-\sqrt{5}, \sqrt{3}]$

(D) $[-\sqrt{5}, -\sqrt{3}] \cup [\sqrt{3}, \sqrt{5}]$

Answer - (D) $[-\sqrt{5}, -\sqrt{3}] \cup [\sqrt{3}, \sqrt{5}]$

Q14 - The value of $tan^{-1}\left(tan\frac{5\pi}{6}\right)$ is

(A) $-\frac{\pi}{6}$

(B) $\frac{\pi}{6}$

(C) $\frac{5\pi}{6}$

(D) $-\frac{5\pi}{6}$

Answer – (A) $-\frac{\pi}{6}$

Q15 - If $y = \frac{1}{a}cot^{-1}\left(\frac{x}{a}\right)$, then $\frac{dy}{dx}$ is equal to

(A) $\frac{1}{x^2+a^2}$

(B) $\frac{1}{a^2-x^2}$

(C) $\frac{1}{\sqrt{x^2+a^2}}$

(D) $\frac{-1}{x^2+a^2}$

Answer - (D) $\frac{-1}{x^2+a^2}$

Q16 - Interval in which the function $f(x) = 2x^2 - 4x + 15$ is increasing, is

(A) $(-1, \infty)$

(B) $(1, \infty)$

(C) $(0, 1)$

(D) $(1, 4)$

Answer – (B) $(1, \infty)$

Q17 – If $|\vec{a}| = 10, |\vec{b}| = 2, |\vec{a} \cdot \vec{b}| = 12$, then $|\vec{a} \times \vec{b}|$ is

(A) 5

(B) 10

(C) 14

(D) 16

Answer - (D) 16

Q18 – Angle between the plane $x + 2y + x = 0$ and $2x - y + z = 15$ is

(A) $tan^{-1}\frac{1}{6}$

(B) $sin^{-1}\frac{1}{6}$

(C) $cos^{-1}\frac{1}{6}$

(D) $\frac{\pi}{6}$

Answer - (C) $cos^{-1}\frac{1}{6}$

Q19 – Vector equation of a line through the point $(2, -1, 4)$ and parallel to the vector $\hat{\imath} + 2\hat{\jmath} - \hat{k}$ is

(A) $\vec{r} = \hat{\imath} + 2\hat{\jmath} - \hat{k} + \lambda(2\hat{\imath} - \hat{\jmath} + 4\hat{k})$

(B) $\vec{r} = \hat{\imath} + 2\hat{\jmath} - \hat{k}$

(C) $\vec{r} = \hat{\imath} - 2\hat{\jmath} - \hat{k}$

(D) $\vec{r} = 2\hat{\imath} - \hat{\jmath} + 4\hat{k} + \lambda(\hat{\imath} + 2\hat{\jmath} - \hat{k})$

Answer - (D) $\vec{r} = 2\hat{\imath} - \hat{\jmath} + 4\hat{k} + \lambda(\hat{\imath} + 2\hat{\jmath} - \hat{k})$

Q20 – If $\vec{a} + \vec{b} = \hat{\imath}$ and $\vec{a} = 2\hat{\imath} - 2\hat{\jmath} + 2\hat{k}$, then $|\vec{b}|$ is equal to

(A) $\frac{1}{3}$

(B) 3

(C) 1

(D) -3

Answer - (B) 3

Q21 - Fill in the blanks :

(i) The eccentricity of the ellipse $3x^2 + 4y^2 = 12$ is ______.

(ii) Length of major axis of the ellipse $5x^2 + 6y^2 = 3$ is _______.

Answer – (a) $\frac{1}{2}$

(b) $\frac{2\sqrt{15}}{5}$

Q22 - Match Column I statement with the right option of Column II :

Column I	Column II
(i) $cos^{-1}\left(\frac{1}{2}sin\left(\frac{\pi}{3} + sin^{-1}\frac{1}{2}\right)\right)$	(a) $\frac{\pi}{3}$
(ii) $cos^{-1}\left(-sin\frac{7\pi}{6}\right)$	(b) $\frac{2\pi}{3}$

Answer –

(i) $cos^{-1}\left(\frac{1}{2}sin\left(\frac{\pi}{3}+sin^{-1}\frac{1}{2}\right)\right) \quad \longrightarrow \quad$ (a) $\frac{\pi}{3}$

(ii) $cos^{-1}\left(-sin\frac{7\pi}{6}\right) \quad \longrightarrow \quad$ (a) $\frac{\pi}{3}$

Q23 - Write True for correct statement and False for incorrect statement:

(i) $\begin{vmatrix} a+b & c+d \\ e+f & g+h \end{vmatrix} = \begin{vmatrix} a & c \\ e & g \end{vmatrix} + \begin{vmatrix} b & d \\ f & h \end{vmatrix}$

(b) A and B are two non-singular square matrix, then $Adj\ A = Adj\ B$ implies $A = B$.

Answer – (a) False

(b) False

Q24 - Write the contrapositive statement of each of the following statements:

(a) If $ABCD$ is a square, then it is rhombus.

(b) If a and b are rational number, then ab is a real number.

Answer – (a) If ABCD is not a rhombus, then it is not a square.

(b) If ab is not a real number, then a and b are not rational numbers.

Q25 – A and B are two 2×2 matrix such that $a_{ij} = i + j$ and $b_{ij} = i - j$.

Match Column I statement with the right option of Column II :

Column I	Column II
(i) $A + 2B$	(a) $\begin{pmatrix} 3 & -2 \\ 4 & -3 \end{pmatrix}$
(ii) AB	(b) $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$
(iii) A'	(c) $\begin{pmatrix} 2 & 1 \\ 5 & 4 \end{pmatrix}$
(iv) B^{-1}	(d) $\begin{pmatrix} 2 & 3 \\ 3 & 4 \end{pmatrix}$

Answer –

(i) $A + 2B$ $\longrightarrow$ (c) $\begin{pmatrix} 2 & 1 \\ 5 & 4 \end{pmatrix}$

(ii) AB $\longrightarrow$ (a) $\begin{pmatrix} 3 & -2 \\ 4 & -3 \end{pmatrix}$

(iii) A' $\longrightarrow$ (d) $\begin{pmatrix} 2 & 3 \\ 3 & 4 \end{pmatrix}$

(iv) B^{-1} $\longrightarrow$ (b) $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

Q26 - Fill in the blanks :

(i) Vector equation of a plane passing through $(1, 4, 6)$ and normal to the vector $\vec{\imath} - 2\vec{\jmath} + \vec{k}$ is ______.

Answer – $r \cdot (\hat{\imath} - 2\hat{\jmath} + \hat{k}) = -1$

(ii) Sum of the intercept from co-ordinate axes made by the plane $2x + 3y + 4z = 24$ is ______.

Answer – 26

(iii) Vector equation of a plane passing through three points whose position vectors are $\vec{a}, \vec{b}, \vec{c}$ is _____.

Answer – $(\vec{r} - \vec{a}) \cdot [(\vec{b} - \vec{a}) \times (\vec{c} - \vec{a})] = 0$

(iv) Perpendicular distance drawn from the point $(-1, 2, -3)$ to the plane $2x - y + 2z = 13$ is ______.

Answer – $23/3\ units$

Q27 - Write True for correct statement and False for incorrect statement:

(i) Tangent to the curve $y = 2x^3 - 15x^2 + 36x - 21$ at $(1, 2)$ is parallel to x-axis.

Answer – False

(ii) $\int x\cos x dx$ is equal to $x\cos x + \sin x + c$.

Answer – False

(iii) $\int e^x \left(2\log x + \frac{2}{x}\right) dx$ is equal to $e^x \log x^2 + c$.

Answer – True

(iv) Normal to the parabola at its vertex is passes through the focus of the parabola.

Answer – True

Q28 - Fill in the blanks :

(i) Degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^{1/3} = x$ is _______.

Answer – 1

(ii) The differential equation for which $y = a\cos x + b\sin x$ (a, b are arbitary parameter) is a solution, is ______.

Answer – $\frac{d^2y}{dx^2} + y = 0$

(iii) The solution of the differential equation $\frac{dy}{dx} = e^{x+y}$ is $e^y =$______.

Answer – $e^y = \frac{1}{C-e^x}$

(iv) Area of the region bounded by the curve $y = x^3$ and $x = 0, x = 1, y = 0$ is _____.

Answer – $1/4\ sq.units$

Q29 - A company produces x unit of output at a total cost of $C = \frac{1}{3}x^3 = 18x^2 + 160x$. The Average Cost (AC) is the cost per unit and the Marginal Cost (MC) is the rate of change of C with respect x. C is minimum at x if its first order derivative is vanished at x; and second order derivative is positive at that point. Similarly, C is maximum at x if first order derivative is vanished at x and second order derivative is negative at that point. Based on above information answer the following questions:

(i) The Average Cost (AC) is given by

(A) $\frac{1}{3}x^2 - 18x + 160$ **(B)** $x^2 - 36x + 160$

(C) $\frac{1}{3}x^3 - 18x + 160$ **(D)** $\frac{1}{3}x^4 - 18x^2 + 160$

Answer – (A) $\frac{1}{3}x^2 - 18x + 160$

(ii) The Marginal Cost (MC) is given by

(A) $3x^2 - 36x + 160$ **(B)** $x^2 - 36x + 160$

(B) $x^2 - 16x + 160$ **(D)** $x^2 + 36x + 160$

Answer – (B) $x^2 - 36x + 160$

(iii) The output at which average cost is equal to marginal cost, is

(A) $27\ units$ | **(B)** $18\ units$

(C) $9\ units$ | **(D)** $36\ units$

Answer – (A) $27\ units$

(iv) The output at which marginal cost is minimum, is

(A) $27\ units$ | **(B)** $18\ units$

(C) $16\ units$ | **(D)** $12\ units$

Answer – (B) $18\ units$

(v) The output at which average cost is minimum, is

(A) $27\ units$ | **(B)** $18\ units$

(C) $9\ units$ | **(D)** $12\ units$

Answer – (A) $27\ units$

(vi) The minimum value of marginal cost is

(A) 164 | **(B)** -164

(C) 216 | **(D)** -216

Answer – (B) -164

SECTION - B

Q30 - Find the area of the triangle whose vertices are $(1, 2), (-2, 3)$ and $(-3, -4)$.

Answer – Given the vertices of the triangle are:

$$A(1,2), B(-2,3), C(-3,-4)$$

Using the area formula:

$$\text{Area of } \triangle ABC = \frac{1}{2}\begin{vmatrix} 1 & 2 & 1 \\ -2 & 3 & 1 \\ -3 & -4 & 1 \end{vmatrix}$$

$$= \frac{1}{2}[\,1\{3-(-4)\} - 2\{(-2)-(-3)\} + 1\{(-2)(-4) - 3(-3)\}]$$

$$= \frac{1}{2}[1(7) - 2(1) + (8+9)]$$

$$= \frac{1}{2}(7 - 2 + 17) = \frac{1}{2}(22) = 11$$

$$\boxed{\therefore \text{ Area of the triangle} = 11 \text{ square units.}}$$

Q31 - Find the equation of circle which touches y-axis at origin and radius is 3 units.

Answer – Since the circle **touches the y-axis at the origin**, the point of tangency is (0, 0).

Since the circle touches the y-axis at origin, the centre must lie on the **x-axis** (perpendicular from centre to tangent passes through point of tangency).

Let the centre be **(a, 0)**.

Since radius = 3 units:

$$|a| = 3 \Rightarrow a = 3 \text{ or } a = -3$$

So, there are **two possible circles:**

Equation of circle: $(x-a)^2 + (y-0)^2 = r^2$

Case 1: Centre = (3, 0), radius = 3

$$(x-3)^2 + y^2 = 9$$

$$x^2 - 6x + 9 + y^2 = 9$$

$$\boxed{x^2 + y^2 - 6x = 0}$$

Case 2: Centre = (−3, 0), radius = 3

$$(x+3)^2 + y^2 = 9$$

$$x^2 + 6x + 9 + y^2 = 9$$

$$\boxed{x^2 + y^2 + 6x = 0}$$

Hence the required equations of circles are:

$$x^2 + y^2 - 6x = 0 \text{ or } x^2 + y^2 + 6x = 0$$

OR

Find the eccentricity and the co-ordinates of the foci of the ellipse $4x^2 + 9y^2 = 1$.

Answer – Given equation of the ellipse:

$$4x^2 + 9y^2 = 1$$

Write it in the standard form:

$$\frac{x^2}{\frac{1}{4}} + \frac{y^2}{\frac{1}{9}} = 1$$

Comparing with the standard equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,$$

we get

$$a^2 = \frac{1}{4}, b^2 = \frac{1}{9}.$$

Since $a^2 > b^2$, the major axis is along the x-axis.

The eccentricity of the ellipse is

$$e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{\frac{1}{9}}{\frac{1}{4}}} = \sqrt{1 - \frac{4}{9}} = \sqrt{\frac{5}{9}} = \frac{\sqrt{5}}{3}.$$

Also,

$$ae = \frac{1}{2} \times \frac{\sqrt{5}}{3} = \frac{\sqrt{5}}{6}.$$

Hence, the coordinates of the foci are

$$\boxed{\left(\frac{\sqrt{5}}{6}, 0\right) \text{ and } \left(-\frac{\sqrt{5}}{6}, 0\right)}.$$

$$\boxed{\text{Eccentricity } = \frac{\sqrt{5}}{3}}$$

$$\boxed{\text{Foci } = \left(\pm\frac{\sqrt{5}}{6}, 0\right)}$$

Q32 - Determine a point on x-axis which is equidistant from two points $A(2,0,0)$ and $B(0,3,0)$.

Answer – Let the required point on the x-axis be

$$P(x,0,0).$$

Since P is equidistant from A and B,

$$PA = PB.$$

Using the distance formula in three dimensions,

$$PA = \sqrt{(x-2)^2 + (0-0)^2 + (0-0)^2} = |x-2|$$

and

$$PB = \sqrt{(x-0)^2 + (0-3)^2 + (0-0)^2} = \sqrt{x^2+9}.$$

Therefore,

$$|x-2| = \sqrt{x^2+9}.$$

Squaring both sides,

$$(x-2)^2 = x^2 + 9$$

$$x^2 - 4x + 4 = x^2 + 9$$

$$-4x = 5$$

$$x = -\frac{5}{4}.$$

Hence, the required point is

$$\boxed{\left(-\frac{5}{4}, 0, 0\right)}.$$

$\therefore$ The point on the x-axis equidistant from $A(2,0,0)$ and $B(0,3,0)$ is $\left(-\frac{5}{4},0,0\right)$.

OR

Find the equation of the plane passing through the point $(1, 2, 1)$ and perpendicular to x-axis.

Answer – Given,

- Point through which the plane passes: $(1, 2, 1)$
- The plane is **perpendicular to the x-axis**.

A plane perpendicular to the x-axis has its **normal parallel to the x-axis**. Therefore, the equation of such a plane is of the form

$$x = a.$$

Since the plane passes through the point $(1, 2, 1)$,

$$a = 1.$$

Hence, the required equation of the plane is

$$\boxed{x = 1.}$$

$$\boxed{\therefore \text{ The equation of the required plane is } x = 1.}$$

Q33 – If vectors $\vec{a} = 2\hat{\imath} - 3\hat{\jmath}$ and $\vec{b} = -6\hat{\imath} + m\hat{\jmath}$ are collinear, then find the value of m.

Answer – Given,

$$\vec{a} = 2\hat{\imath} - 3\hat{\jmath}, \vec{b} = -6\hat{\imath} + m\hat{\jmath}$$

Since the vectors are **collinear**, their corresponding components are proportional.

$$\frac{2}{-6} = \frac{-3}{m}$$

Cross-multiplying,

$$2m = (-6)(-3)$$

$$2m = 18$$

$$m = 9$$

$$\boxed{\therefore\ m = 9.}$$

Q34 – If $A = \begin{bmatrix} \sin x & 1 & 1 \\ 1 & e^x & 1 \\ \cos x & 0 & 1 \end{bmatrix}$, then show that $|4A| = 64|A|$.

Answer – Given,

$$A = \begin{bmatrix} \sin x & 1 & 1 \\ 1 & e^x & 1 \\ \cos x & 0 & 1 \end{bmatrix}$$

Multiplying each element of the matrix by 4, we get

$$4A = \begin{bmatrix} 4\sin x & 4 & 4 \\ 4 & 4e^x & 4 \\ 4\cos x & 0 & 4 \end{bmatrix}.$$

For a matrix of order 3×3, if each element is multiplied by a constant k, then its determinant is multiplied by k^3.

Hence,

$$|kA| = k^3|A|.$$

Putting $k = 4$,

$$|4A| = 4^3|A| = 64|A|.$$

$$\boxed{|4A| = 64|A|.}$$

Hence proved.

Q35 – For what values of $x$and y, the following matrices are equal?

$$A = \begin{bmatrix} 2x+1 & 3y \\ 0 & y^2 - 5y \end{bmatrix}, B = \begin{bmatrix} x+3 & y^2+2 \\ 0 & -6 \end{bmatrix}$$

Answer – Since the matrices $A$and $B$are equal, their corresponding elements must be equal.

Comparing the corresponding elements,

1. $2x + 1 = x + 3$

$$2x - x = 3 - 1$$

$$x = 2$$

2. $3y = y^2 + 2$

$$y^2 - 3y + 2 = 0$$

$$(y-1)(y-2)=0$$

$$\therefore\ y=1 \text{ or } y=2$$

3. $y^2-5y=-6$

$$y^2-5y+6=0$$

$$(y-2)(y-3)=0$$

$$\therefore\ y=2 \text{ or } y=3$$

The common value satisfying both conditions is

$$\boxed{y=2.}$$

Hence,

$$\boxed{x=2, y=2.}$$

$$\boxed{\therefore \text{ The required values are } x=2 \text{ and } y=2.}$$

Q36 – Solve for x, $\cos\left(\tan^{-1}x+\cot^{-1}\sqrt{3}\right)=0$.

Answer – Given,

$$\cos\left(\tan^{-1}x+\cot^{-1}\sqrt{3}\right)=0$$

Since

$$\cot^{-1}\sqrt{3}=30^\circ=\frac{\pi}{6},$$

the equation becomes

$$\cos\left(\tan^{-1}x+\frac{\pi}{6}\right)=0.$$

Now,

$$\cos\theta=0$$

when

$$\theta=\frac{\pi}{2}.$$

Therefore,

$$\tan^{-1} x + \frac{\pi}{6} = \frac{\pi}{2}$$

$$\tan^{-1} x = \frac{\pi}{2} - \frac{\pi}{6} = \frac{\pi}{3}.$$

Taking tangent on both sides,

$$x = \tan\frac{\pi}{3} = \sqrt{3}.$$

$$\boxed{\therefore\ x = \sqrt{3}.}$$

Q37 – If $y = e^x \log(1 + x^2)$, then find $\frac{dy}{dx}$.

Answer - Given,

$$y = e^x \log(1 + x^2)$$

Using the **product rule**,

$$\frac{dy}{dx} = \frac{d}{dx}(e^x) \cdot \log(1 + x^2) + e^x \cdot \frac{d}{dx}[\log(1 + x^2)].$$

$$\frac{d}{dx}(e^x) = e^x$$

and

$$\frac{d}{dx}[\log(1 + x^2)] = \frac{1}{1 + x^2} \cdot 2x = \frac{2x}{1 + x^2}.$$

Therefore,

$$\frac{dy}{dx} = e^x \log(1 + x^2) + e^x \left(\frac{2x}{1 + x^2}\right).$$

Taking e^xas the common factor,

$$\boxed{\frac{dy}{dx} = e^x \left(\log(1 + x^2) + \frac{2x}{1 + x^2}\right).}$$

$$\boxed{\therefore\ \frac{dy}{dx} = e^x \left(\log(1 + x^2) + \frac{2x}{1 + x^2}\right).}$$

OR

Find : $\lim_{x\to\pi}\frac{1+\cos x}{\tan^2 x}$

Answer – Given,

$$\lim_{x\to\pi}\frac{1+\cos x}{\tan^2 x}$$

Using the identity,

$$\tan^2 x=\frac{\sin^2 x}{\cos^2 x},$$

we get

$$=\lim_{x\to\pi}\frac{(1+\cos x)\cos^2 x}{\sin^2 x}.$$

Now, using

$$\sin^2 x=(1-\cos x)(1+\cos x),$$

we have

$$=\lim_{x\to\pi}\frac{(1+\cos x)\cos^2 x}{(1-\cos x)(1+\cos x)}$$

$$=\lim_{x\to\pi}\frac{\cos^2 x}{1-\cos x}.$$

Substituting $x=\pi$,

$$\cos\pi=-1.$$

Hence,

$$=\frac{(-1)^2}{1-(-1)}=\frac{1}{2}.$$

$$\boxed{\therefore\ \lim_{x\to\pi}\frac{1+\cos x}{\tan^2 x}=\frac{1}{2}.}$$

Q38 - State Rolle's theorem for the function $f(x)$in $[a,b]$.

Answer – Rolle's Theorem:

If a function $f(x)$:

1. is **continuous** on the closed interval $[a,b]$,
2. is **differentiable** on the open interval (a,b), and
3. satisfies $f(a)=f(b)$,

then there exists at least one point $c$in the interval (a,b) such that

$$f(a)=f(b) \implies \exists\, c \in (a,b): f'(c)=0$$

$$\boxed{f'(c)=0.}$$

Thus, there is at least one point between $a$and $b$where the tangent to the curve is parallel to the x-axis.

OR

Find the intervals in which the function $f(x)=2x^3+x^2-20x$ is decreasing.

Answer – Given,

$$f(x)=2x^3+x^2-20x$$

Differentiate with respect to x:

$$\begin{aligned} f'(x) &= 6x^2+2x-20 \\ &= 2(3x^2+x-10) \\ &= 2(3x-5)(x+2) \end{aligned}$$

To find the interval where the function is decreasing, we require

$$f'(x)<0.$$

Since $2>0$,

$$(3x-5)(x+2)<0.$$

The critical points are

$$x=-2, x=\frac{5}{3}.$$

Now, checking the sign of $f'(x)$:

- For $x < -2$, $f'(x) > 0$.
- For $-2 < x < \frac{5}{3}$, $f'(x) < 0$.
- For $x > \frac{5}{3}$, $f'(x) > 0$.

Hence, the function is decreasing on

$$\boxed{\left(-2,\ \frac{5}{3}\right).}$$

$\therefore$ The function $f(x) = 2x^3 + x^2 - 20x$ is decreasing in the interval $\left(-2, \frac{5}{3}\right)$.

Q39 - Solve for x, y, z from the following system of linear equations, using matrix method :

$\mathbf{x + 2y + 3z = 0}$

$\mathbf{2x + 3y + 2z = -2}$

$\mathbf{3x - 3y + 4z = 13}$

Answer – The given system of equations can be written in the matrix form as

$$AX = B,$$

where

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 2 \\ 3 & -3 & 4 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 0 \\ -2 \\ 13 \end{bmatrix}.$$

Since

$$X = A^{-1}B,$$

first find the determinant of A.

$$|A| = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 2 \\ 3 & -3 & 4 \end{vmatrix}$$

$$= 1(12 + 6) - 2(8 - 6) + 3(-6 - 9)$$

$$= 18 - 4 - 45 = -31.$$

Since

$$|\ A\ | = -31 \neq 0,$$

the inverse of A exists.

The adjoint of $A$is

$$\operatorname{adj}(A)=\begin{bmatrix}18 & -17 & -5\\ -2 & -5 & 4\\ -15 & 9 & -1\end{bmatrix}.$$

Hence,

$$A^{-1}=\frac{1}{-31}\begin{bmatrix}18 & -17 & -5\\ -2 & -5 & 4\\ -15 & 9 & -1\end{bmatrix}.$$

Now,

$$X=A^{-1}B=\frac{1}{-31}\begin{bmatrix}18 & -17 & -5\\ -2 & -5 & 4\\ -15 & 9 & -1\end{bmatrix}\begin{bmatrix}0\\ -2\\ 13\end{bmatrix}.$$

Multiplying,

$$=\frac{1}{-31}\begin{bmatrix}-31\\ 62\\ -31\end{bmatrix}=\begin{bmatrix}1\\ -2\\ 1\end{bmatrix}.$$

Therefore,

$$\boxed{x=1, y=-2, z=1.}$$

Verification:

$$1+2(-2)+3(1)=0,$$
$$2(1)+3(-2)+2(1)=-2,$$
$$3(1)-3(-2)+4(1)=13.$$

Hence, the solution is verified.

$$\boxed{\therefore\ x=1,\ y=-2,\ z=1.}$$

Q40 - Find the equation of a line passing through the intersection of lines $5x-3y=1$ and $2x+3y=23$ and perpendicular to the line $5x+3y=1$.

Answer – The required line passes through the point of intersection of

$$5x-3y=1$$

and

$$2x + 3y = 23.$$

Adding the two equations,

$$5x - 3y + 2x + 3y = 1 + 23$$

$$7x = 24$$

$$x = \frac{24}{7}.$$

Substituting in

$$2x + 3y = 23,$$

we get

$$2\left(\frac{24}{7}\right) + 3y = 23$$

$$\frac{48}{7} + 3y = 23$$

$$3y = \frac{161 - 48}{7} = \frac{113}{7}$$

$$y = \frac{113}{21}.$$

Hence, the point of intersection is

$$\left(\frac{24}{7}, \frac{113}{21}\right).$$

Now, the line

$$5x + 3y = 1$$

can be written as

$$y = -\frac{5}{3}x + \frac{1}{3}.$$

Therefore, its slope is

$$m_1 = -\frac{5}{3}.$$

The slope of a line perpendicular to it is

$$m_2 = \frac{3}{5}.$$

Using the point-slope form,

$$y - \frac{113}{21} = \frac{3}{5}\left(x - \frac{24}{7}\right).$$

Multiplying throughout by 105,

$$105y - 565 = 63x - 216.$$

Hence,

$$63x - 105y + 349 = 0.$$

∴ The required equation of the line is :

$$\boxed{63x - 105y + 349 = 0.}$$

OR

Find the equation of the circle passing through the points $(1,0)$, $(0,-6)$ and $(3,4)$.

Answer – Let the equation of the circle be

$$x^2 + y^2 + 2gx + 2fy + c = 0.$$

Since the circle passes through $(1,0)$,

$$1 + 2g + c = 0$$
$$\boxed{2g + c = -1} \cdots (1)$$

Since it passes through $(0,-6)$,

$$36 - 12f + c = 0$$
$$\boxed{c = 12f - 36} \cdots (2)$$

Since it passes through $(3,4)$,

$$9 + 16 + 6g + 8f + c = 0$$
$$\boxed{25 + 6g + 8f + c = 0} \cdots (3)$$

From (1),

$$c = -1 - 2g.$$

Substituting in (3),

$$25 + 6g + 8f - 1 - 2g = 0$$
$$24 + 4g + 8f = 0$$
$$\boxed{g + 2f = -6} \cdots (4)$$

Also, from (1) and (2),

$$-1 - 2g = 12f - 36$$
$$35 - 2g = 12f.$$

Substituting the value of f from (4),

$$g + \frac{35 - 2g}{6} = -6$$
$$6g + 35 - 2g = -36$$
$$4g = -71$$
$$g = -\frac{71}{4}.$$

Hence,

$$f = \frac{35 - 2g}{12} = \frac{35 + \frac{71}{2}}{12} = \frac{47}{8}.$$

Also,

$$c = -1 - 2g = -1 + \frac{71}{2} = \frac{69}{2}.$$

Therefore, the required equation of the circle is

$$x^2 + y^2 - \frac{71}{2}x + \frac{47}{4}y + \frac{69}{2} = 0.$$

Multiplying throughout by 4,

$$\boxed{4x^2 + 4y^2 - 142x + 47y + 138 = 0.}$$

$\boxed{\therefore \text{ The required equation of the circle is } 4x^2 + 4y^2 - 142x + 47y + 138 = 0.}$

Q41 - Show that the function $f:\mathbb{R}\to\mathbb{R}$ defined by $f(x)=x^2-1$ is neither one-one nor onto function.

Answer – Given,

$$f:\mathbb{R}\to\mathbb{R}, f(x)=x^2-1.$$

(i) To show that f is not one-one

Take two distinct real numbers,

$$x=1 \text{ and } x=-1.$$

Then,

$$f(1)=1^2-1=0$$

and

$$f(-1)=(-1)^2-1=0.$$

Thus,

$$f(1)=f(-1),$$

although

$$1\neq -1.$$

Hence, different elements of the domain have the same image.

$$\boxed{\therefore\ f \text{ is not one-one.}}$$

(ii) To show that f is not onto

For every real number x,

$$x^2\geq 0.$$

Therefore,

$$f(x)=x^2-1\geq -1.$$

Hence, no real number less than -1 is the image of any element of $\mathbb{R}$.

For example,

$$-2 \in \mathbb{R},$$

but there is no real number $x$such that

$$x^2 - 1 = -2.$$

This gives

$$x^2 = -1,$$

which has no real solution.

Therefore, -2 has no pre-image.

$$\boxed{\therefore\ f \text{ is not onto.}}$$

Hence,

$$\boxed{\therefore\ f(x) = x^2 - 1 \text{ is neither one-one nor onto.}}$$

Q42 – Let the function $f(x)$is defined by

$$f(x) = \begin{cases} \dfrac{1 - \cos 4x}{x^2}, & x < 0 \\ a, & x = 0 \\ \dfrac{\sqrt{x}}{\sqrt{16 + \sqrt{x}} - 4}, & x > 0 \end{cases}$$

is continuous at $x = 0$. Determine a.

Answer – For continuity at $x = 0$,

$$\lim_{x \to 0^-} f(x) = f(0) = \lim_{x \to 0^+} f(x) = a.$$

Left-hand limit

$$\lim_{x \to 0^-} \frac{1 - \cos 4x}{x^2}$$

Using the standard limit,

$$\lim_{\theta \to 0} \frac{1 - \cos \theta}{\theta^2} = \frac{1}{2},$$

we get

$$= \lim_{x\to 0} \frac{1-\cos 4x}{(4x)^2} \times 16$$

$$= \frac{1}{2} \times 16 = 8.$$

Right-hand limit

$$\lim_{x\to 0^+} \frac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4}$$

Rationalising the denominator,

$$= \lim_{x\to 0^+} \frac{\sqrt{x}\left(\sqrt{16+\sqrt{x}}+4\right)}{(16+\sqrt{x})-16}$$

$$= \lim_{x\to 0^+} \left(\sqrt{16+\sqrt{x}}+4\right)$$

$$= 4+4 = 8.$$

Since

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = 8,$$

for continuity,

$$\boxed{\therefore\ a = 8.}$$

OR

At what points, tangents to the curve $y = 2x^3 - 15x^2 + 36x - 21$ be parallel to x-axis. Also, find the equation of the tangents to the curve at these points.

Answer – Given,

$$y = 2x^3 - 15x^2 + 36x - 21.$$

Differentiate with respect to x:

$$\frac{dy}{dx} = 6x^2 - 30x + 36.$$

$$= 6(x^2 - 5x + 6)$$

$$= 6(x-2)(x-3).$$

A tangent is parallel to the x-axis when

$$\frac{dy}{dx} = 0.$$

Therefore,

$$6(x-2)(x-3) = 0.$$

Hence,

$$x = 2 \text{or} x = 3.$$

Now, find the corresponding values of y.

For $x = 2$,

$$y = 2(2)^3 - 15(2)^2 + 36(2) - 21$$
$$= 16 - 60 + 72 - 21 = 7.$$

So, one point is (2,7).

For $x = 3$,

$$y = 2(3)^3 - 15(3)^2 + 36(3) - 21$$
$$= 54 - 135 + 108 - 21 = 6.$$

So, the other point is (3,6).

Thus, the tangents are parallel to the x-axis at

$$\boxed{(2,7) \text{ and } (3,6).}$$

Since the slope of each tangent is zero, their equations are

$$\boxed{y = 7}$$

and

$$\boxed{y = 6.}$$

∴ The tangents are parallel to the $x - axis$ at (2,7) and (3,6), and their equations are $y = 7$ and $y = 6$.

Q43 - Find the angle between the lines $x-2y+z=0=x+2y-2z$ and $\frac{x-1}{3}=\frac{1-y}{1}=\frac{z}{0}$.

Answer – The first line is the line of intersection of the planes

$$x-2y+z=0$$

and

$$x+2y-2z=0.$$

The normals to these planes are

$$\overrightarrow{n_1}=(1,-2,1), \overrightarrow{n_2}=(1,2,-2).$$

The direction vector of the line is

$$\overrightarrow{d_1}=\overrightarrow{n_1}\times\overrightarrow{n_2}.$$

$$\overrightarrow{d_1}=\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 1 & -2 & 1\\ 1 & 2 & -2\end{vmatrix}$$

$$=(4-2)\hat{\imath}-(-2-1)\hat{\jmath}+(2+2)\hat{k}$$

$$=2\hat{\imath}+3\hat{\jmath}+4\hat{k}.$$

Hence, the direction ratios of the first line are $(2,3,4)$.

The second line is

$$\frac{x-1}{3}=\frac{1-y}{1}=\frac{z}{0}.$$

Since

$$\frac{1-y}{1}=\frac{y-1}{-1},$$

its direction ratios are $(3,-1,0)$.

Let θbe the angle between the two lines.

Using the formula,

$$\cos\theta=\frac{l_1l_2+m_1m_2+n_1n_2}{\sqrt{l_1^2+m_1^2+n_1^2}\sqrt{l_2^2+m_2^2+n_2^2}}.$$

Substituting the values,

$$\cos\theta = \frac{2(3)+3(-1)+4(0)}{\sqrt{2^2+3^2+4^2}\sqrt{3^2+(-1)^2+0^2}}$$

$$= \frac{6-3}{\sqrt{29}\sqrt{10}} = \frac{3}{\sqrt{290}}.$$

Hence,

$$\boxed{\theta = \cos^{-1}\left(\frac{3}{\sqrt{290}}\right).}$$

Therefore,

$$\boxed{\therefore \text{ The angle between the given lines is } \cos^{-1}\left(\frac{3}{\sqrt{290}}\right).}$$

Q44 - Solve the following LPP graphically:

Maximize

$\boldsymbol{Z = 3x + 4y}$

Subject to constraints

$\boldsymbol{2x + y \leq 1000}$
$\boldsymbol{x + y \leq 800}$
$\boldsymbol{x \leq 400}$
$\boldsymbol{y \leq 700}$
$\boldsymbol{x \geq 0,\ y \geq 0}$

Answer – The given constraints are:

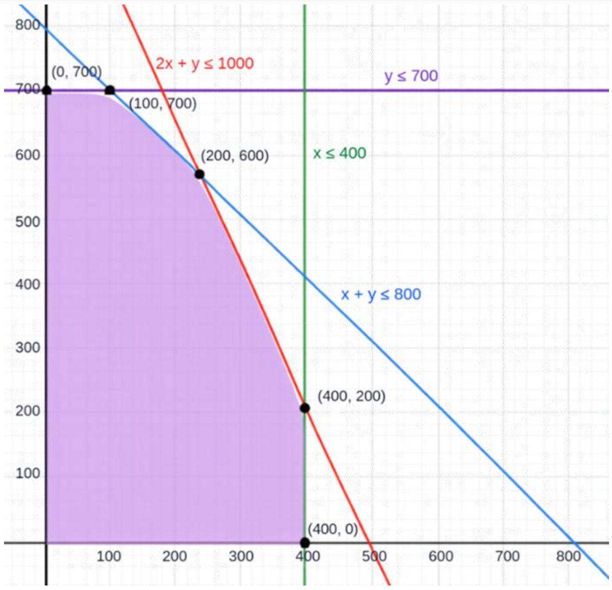
$$2x + y \leq 1000,$$

$$x + y \leq 800,$$

$$x \leq 400,$$

$$y \leq 700,$$

$$x \geq 0,\ y \geq 0.$$



The corner points of the feasible region are obtained as follows:

Corner Point	**Coordinates**	**Calculation**
O	$(0,0)$	Origin
A	$(400,0)$	$x = 400,\ y = 0$
B	$(400,200)$	$x = 400,\ 2x + y = 1000$
C	$(200,600)$	Solving $2x + y = 1000$and $x + y = 800$
D	$(100,700)$	$y = 700,\ x + y = 800$
E	$(0,700)$	$x = 0,\ y = 700$

Now, calculate the value of the objective function

$$Z = 3x + 4y.$$

Corner Point	$Z = 3x + 4y$
$(0,0)$	0
$(400,0)$	1200
$(400,200)$	2000
$(200,600)$	3000
$(100,700)$	3100
$(0,700)$	2800

The maximum value of Z is 3100.

at the point $(100,700)$.

Answer:

$$\boxed{\text{Maximum value of } Z = 3100}$$

$$\boxed{\text{at } (x, y) = (100,\ 700).}$$

OR

There are two types of fertilisers A and B. A consists of 12% nitrogen and 5% phosphoric acid; whereas B consists of 4% nitrogen and 5% phosphoric acid. After testing the soil conditions, farmer finds that he needs at least 12 kg of nitrogen and 12 kg of phosphoric acid for his crops. If A costs ₹110 per kg and B costs ₹48 per kg. Formulate the problem as a linear programming problem and graphically determine how much of each type of fertiliser should be used so that the nutrient requirements are met at minimum cost?

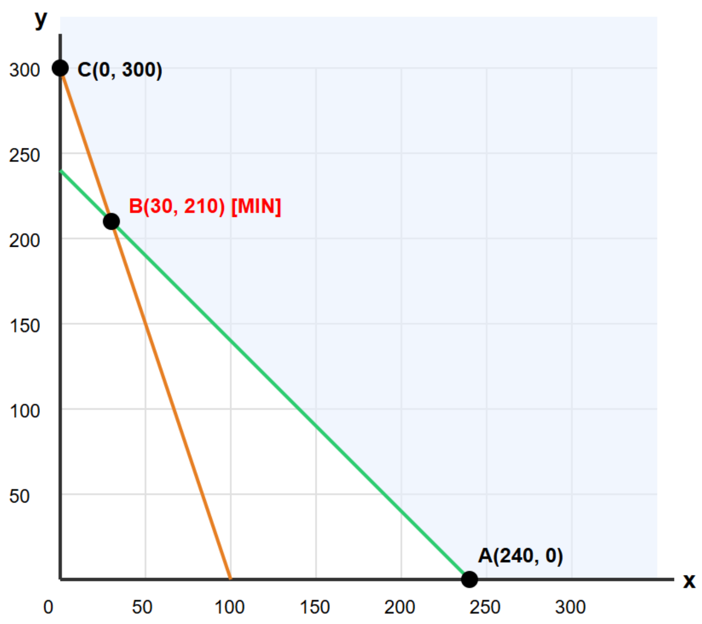
Answer - Let

- x= quantity (in kg) of fertiliser A,
- y= quantity (in kg) of fertiliser B.

Objective Function

$$\boxed{Z = 110x + 48y}$$

which is to be **minimized**.



Constraints

Nitrogen requirement:

Since fertiliser A contains 12% nitrogen and fertiliser B contains 4% nitrogen,

$$0.12x + 0.04y \geq 12$$

Multiplying by 25,

$$\boxed{3x + y \geq 300.}$$

Phosphoric acid requirement:

Since both fertilisers contain 5% phosphoric acid,

$$0.05x + 0.05y \geq 12$$

Multiplying by 20,

$$\boxed{x + y \geq 240.}$$

Also,

$$\boxed{x \geq 0, y \geq 0.}$$

Corner Points of the Feasible Region

The boundary lines are

$$3x + y = 300$$

and

$$x + y = 240.$$

Their intersection is obtained by solving

$$3x + y = 300$$

$$x + y = 240.$$

Subtracting,

$$2x = 60 \Rightarrow x = 30.$$

Substituting,

$$30 + y = 240$$

$$y = 210.$$

Thus,

$$(30, 210)$$

is one corner point.

The other corner points are

$$(0,300) \text{ and } (240,0).$$

Calculation of Z

Corner Point	$Z = 110x + 48y$
$(0,300)$	$110(0) + 48(300) = 14400$
$(30,210)$	$110(30) + 48(210) = 13380$
$(240,0)$	$110(240) = 26400$

The minimum value of Z is

$$13380$$

at

$$(x, y) = (30,210).$$

Answer

$$\boxed{\text{Fertiliser A} = 30\text{ kg}} \text{ and } \boxed{\text{Fertiliser B} = 210\text{ kg}}$$

$$\boxed{\text{Minimum Cost} = ₹13,380}$$

∴ The farmer should use 30 kg of fertiliser A and 210 kg of fertiliser B to meet the nutrient requirements at the minimum cost.

Q45 - Evaluate the following integral as a limit of sum : $\int_2^4 2^x \, dx$.

Answer – By definition, the integral as a limit of sum is:

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} h \sum_{r=0}^{n-1} f(a+rh)$$

where $h = \frac{b-a}{n}$

Here, $f(x) = 2^x, a = 2, b = 4$

$$h = \frac{4-2}{n} = \frac{2}{n}$$

So as n → ∞, h → 0

Setting up the sum:

$$\int_2^4 2^x\ dx = \lim_{h\to 0} h \sum_{r=0}^{n-1} f(2+rh)$$

$$= \lim_{h\to 0} h \sum_{r=0}^{n-1} 2^{2+rh}$$

$$= \lim_{h\to 0} h \sum_{r=0}^{n-1} 4\cdot 2^{rh}$$

$$= \lim_{h\to 0} 4h \sum_{r=0}^{n-1} (2^h)^r$$

Using Geometric Progression formula $\sum_{r=0}^{n-1} q^r = \frac{q^n-1}{q-1}$, where $q = 2^h$:

$$= \lim_{h\to 0} 4h \cdot \frac{(2^h)^n - 1}{2^h - 1}$$

Since nh = 2:

$$= \lim_{h\to 0} 4h \cdot \frac{2^{nh}-1}{2^h-1} = \lim_{h\to 0} 4h \cdot \frac{2^2-1}{2^h-1}$$

$$= \lim_{h\to 0} \frac{4h}{2^h-1} \times 3$$

$$= 12 \lim_{h\to 0} \frac{h}{2^h-1}$$

Using standard limit: $\lim_{h\to 0} \frac{a^h-1}{h} = \log_e a$, so $\lim_{h\to 0} \frac{h}{2^h-1} = \frac{1}{\log_e 2}$

$$= 12 \times \frac{1}{\log_e 2}$$

$$\boxed{\int_2^4 2^x \, dx = \frac{12}{\log_e 2}}$$

OR

Find the area of the smallest region bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and a line $\frac{x}{a} + \frac{y}{b} = 1$, using integration.

Answer – We need to find the area of the region bounded by:

- Ellipse: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
- Line: $\frac{x}{a} + \frac{y}{b} = 1$

Step 1: Find intersection points

From the line: $\frac{y}{b} = 1 - \frac{x}{a} \rightarrow y = b\left(1 - \frac{x}{a}\right)$

The line meets the axes at **(a, 0)** and **(0, b)**.

The ellipse also passes through **(a, 0)** and **(0, b)**.

So, the two curves intersect at **A = (a, 0)** and **B = (0, b)**.

Step 2: Express y from both curves (in first quadrant)

From ellipse:

$$y_{ellipse} = \frac{b}{a}\sqrt{a^2 - x^2}$$

From line:

$$y_{line} = b\left(1 - \frac{x}{a}\right)$$

Step 3: Identify the smaller region

The smaller region lies **between the line and the ellipse** in the first quadrant, where:

$$y_{ellipse} \geq y_{line} \text{ for } 0 \leq x \leq a$$

Required Area = Area under ellipse − Area under line (from x = 0 to x = a)

$$\text{Area} = \int_0^a y_{ellipse}\ dx - \int_0^a y_{line}\ dx$$

Step 4: Evaluate Area under the line

$$\int_0^a b\left(1-\frac{x}{a}\right)dx = b\left[x-\frac{x^2}{2a}\right]_0^a$$

$$= b\left[a-\frac{a^2}{2a}\right] = b\left[a-\frac{a}{2}\right] = b\cdot\frac{a}{2} = \frac{ab}{2}$$

Step 5: Evaluate Area under the ellipse

$$\int_0^a \frac{b}{a}\sqrt{a^2-x^2}\,dx$$

Using standard formula: $\int_0^a \sqrt{a^2-x^2}\ dx = \frac{\pi a^2}{4}$

$$= \frac{b}{a}\cdot\frac{\pi a^2}{4} = \frac{\pi ab}{4}$$

Step 6: Required Area

$$\text{Area} = \frac{\pi ab}{4} - \frac{ab}{2}$$

$$\boxed{\text{Area} = \frac{ab}{4}(\pi - 2)\text{ square units}}$$

Thank you!

We hope you found this material helpful. We wish you the very best for your examination.

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