NIOS PYQ's SOLUTIONS

PREVIOUS YEARS' QUESTIONS & ANSWERS

APRIL-2024

Your Path to Success

SECTION - A

Q1 - The x-intercept and the y-intercept of the line $4x - 3y + 6 = 0$ are respectively

(A) $-\frac{3}{2}, -2$

(B) $\frac{2}{3}, -\frac{1}{2}$

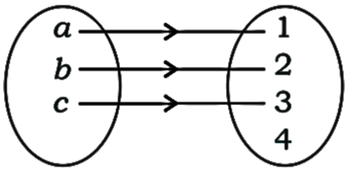
(C) $-\frac{3}{2}, 2$

(D) $-\frac{2}{3}, \frac{1}{2}$

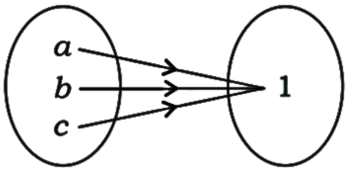
Answer - (C) $-\frac{3}{2}, 2$

Q2 – (a) Which one of the following mappings represents an onto function?

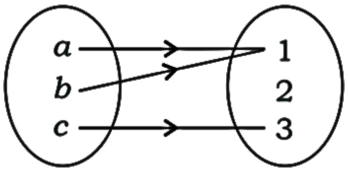
(A)



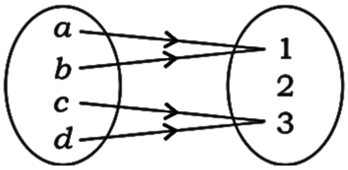
(B)



(C)

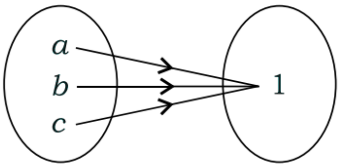


(D)



Answer –

(B)



OR

(b) The principal value of $cos^{-1}\left(-\frac{1}{2}\right)$ $is.$

(A) $-\frac{\pi}{3}$

(B) $\frac{\pi}{3}$

(C) $\frac{2\pi}{3}$

(D) $\frac{4\pi}{3}$

Answer – (C) $\frac{2\pi}{3}$

Q3 – (a) If $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 9 & 2 \\ 18 & 4 \end{vmatrix}$ and x > 0, then the value of x is

(A) 4 **(B) 9**

(C) 6 **(D) 36**

Answer - (C) 6

OR

(b) If $2\begin{bmatrix} x & y \\ 2 & 0 \end{bmatrix} + 3\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 9 & 15 \\ 4 & 6 \end{bmatrix}$ then the value of x and y are

(A) x = 3, y = 9 **(B) x = 9, y = 3**

(C) x = 0, y = 0 **(D) x = 4, y = 8**

Answer – (A) x = 3, y = 9

Q4 – (a) The slope of the line $y + 3 = 0$ is

(A) 0 **(B) 1**

(C) – 3 **(D) – 1**

Answer - (A) 0

OR

(b) The equation of the line having slope $-\frac{3}{2}$ and passing through the point (–1, 2) is

(A) 3x – 2y + 1 = 0 **(B) 3x + 2y + 1 = 0**

(C) 3x – 2y – 1 = 0 **(D) 3x + 2y – 1 = 0**

Answer - (D) 3x + 2y – 1 = 0

Q5 - The slope of the tangent to the curve $\sqrt{x}+\sqrt{y}=1\ at\ \left(\frac{1}{4},\frac{1}{4}\right)$ is

(A) 1 **(B) – 1**

(C) 2 **(D) – ½**

Answer - (B) – 1

Q6 - The direction cosines of y-axis are

(A) 1, 1, 0 **(B) 1, 0, 1**

(C) 0, 1, 0 **(D) 0, 1, 1**

Answer - (C) 0, 1, 0

OR

(b) A unit vector parallel to the resultant of vectors $\vec{a}=3\hat{\imath}-\hat{\jmath}+4\hat{k}$ and $\vec{b}=\hat{\imath}+\hat{\jmath}+\hat{k}$ is

(A) $\frac{4}{\sqrt{45}}\hat{\imath}+\frac{2}{\sqrt{45}}\hat{\jmath}+\frac{5}{\sqrt{45}}\hat{k}$ **(B)** $\frac{4}{\sqrt{45}}\hat{\imath}-\frac{2}{\sqrt{45}}\hat{\jmath}+\frac{5}{\sqrt{45}}\hat{k}$

(C) $\frac{4}{\sqrt{41}}\hat{\imath}+\frac{5}{\sqrt{41}}\hat{k}$ **(D)** $\frac{2}{\sqrt{17}}\hat{\imath}-\frac{2}{\sqrt{17}}\hat{\jmath}+\frac{3}{\sqrt{17}}\hat{k}$

Answer - (C) $\frac{4}{\sqrt{41}}\hat{\imath}+\frac{5}{\sqrt{41}}\hat{k}$

Q7 – If $A=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}5\\-4\\1\end{bmatrix}$ then the value of $x+y+z$ is

(A) 0 **(B) – 2**

(C) 1 **(D) 2**

Answer - (D) 2

Q8 – (a) The interval in which the function $f(x) = sinx, x \in (0, 2\pi)$ is decreasing, is

(A) $\left(0, \frac{\pi}{2}\right)$

(B) $(0, \pi)$

(C) $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

(D) $(\pi, 2\pi)$

Answer - (C) $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

OR

(b) The value of x at which the function $f(x) = 2cosx + x$ has a maximum or minimum, is

(A) $\frac{\pi}{6}$

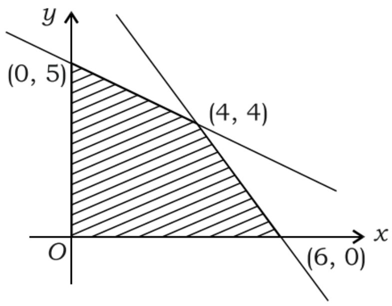
(B) $\frac{\pi}{3}$

(C) $\frac{\pi}{2}$

(D) π

Answer - (A) $\frac{\pi}{6}$

Q9 - (a) The feasible region (shaded) for an LPP is shown in the figure below. Maximum of $Z = 3x + 8y$ occurs at which of the following points?



(A) (0, 5)

(B) (4, 4)

(C) (6, 0) and (0, 5)

(D) At every point of the line segment joining the points (4, 4) and (6, 0)

Answer - (B) (4, 4)

OR

(b) What is the converse of the given statement?

"If n is a positive number, then n^3 is a positive number."

(A) If n is not a positive number, then n^3 is not a positive number.

(B) If n^3 is not a positive number, then n is not a positive number.

(C) If n^3 is a positive number, then n is a positive number.

(D) If n is a positive number, then n^3 is not a positive number.

Answer - (C) If n^3 is a positive number, then n is a positive number.

Q10 - The centre and radius of the circle $x^2 + y^2 + 3x - y = 4$ are respectively

(A) $\left(-\frac{3}{2}, \frac{1}{2}\right), \frac{\sqrt{6}}{2}$

(B) $\left(\frac{3}{2}, -\frac{1}{2}\right), \sqrt{\frac{13}{2}}$

(C) $(-3, 1), \sqrt{14}$

(D) $\left(-\frac{3}{2}, \frac{1}{2}\right), \frac{\sqrt{26}}{2}$

Answer - (D) $\left(-\frac{3}{2}, \frac{1}{2}\right), \frac{\sqrt{26}}{2}$

Q11 - The degree of the following differential equation is $4\left(\frac{d^2y}{dx^2}\right)^3 - 3\left(\frac{d^2y}{dx^2}\right) - 5\left(\frac{dy}{dx}\right)^4 - 1 = 0$

(A) 1

(B) 3

(C) 4

(D) 2

Answer - (B) 3

Q12 - (a) If $A = \begin{bmatrix} cosx & -sinx \\ sinx & cosx \end{bmatrix}$ and $A + A' = I$, then the value of x is

(A) $\frac{\pi}{2}$

(B) $\frac{\pi}{3}$

(C) $\frac{\pi}{4}$

(D) $\frac{\pi}{6}$

Answer - (B) $\frac{\pi}{3}$

OR

(b) If A is a square matrix of order 3 and |A| = 5, then the value of |2A′| is

(A) – 40 **(B) 10**

(C) – 10 **(D) 40**

Answer - (D) 40

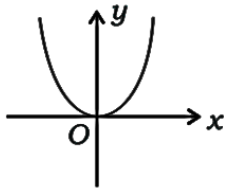
Q13 - The position vectors of the points P, Q and R are $\vec{a} - 2\vec{b}$, $3\vec{a} - 4\vec{b}$ and $7\vec{a} - 8\vec{b}$ respectively. If $\overrightarrow{QR} = \lambda\overrightarrow{PQ}$, then the value of λ is

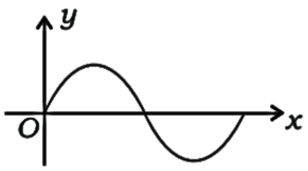
(A) 2 **(B) 4**

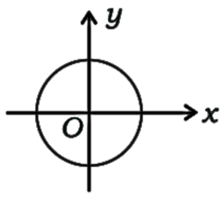
(C) ½ **(D) – 2**

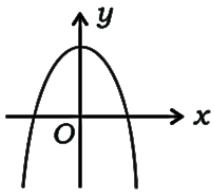
Answer - (A) 2

Q14 - Which one of the following graphs does not represent the graph of a function?

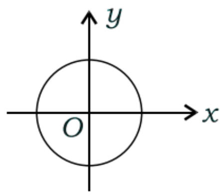
(A) 

(B) 

(C) 

(D) 

Answer –

(C) 

Q15 - (a) The derivative of $sinx$ with respect to $log\ x$ is

(A) $\frac{cosx}{x}$

(B) $xcosx$

(C) $\frac{1}{xcosx}$

(D) $\frac{x}{cosx}$

Answer - (B) $xcosx$

OR

(b) If $y = sin^{-1}(cos\ x)$, then $\frac{dy}{dx} =$

(A) $-\frac{1}{sin\ x}$

(B) $cosec\ x$

(C) 1

(D) -1

Answer - (D) -1

Q16 - If $A = [a_{ij}]$ is a matrix of order 2×3 and $a_{ij} = \frac{i+2j}{2}$, then the matrix A

(A) $\begin{bmatrix} \frac{3}{2} & 3 \\ 2 & 3 \\ \frac{5}{2} & \frac{7}{2} \end{bmatrix}$

(B) $\begin{bmatrix} \frac{3}{2} & 2 \\ \frac{5}{2} & 3 \\ \frac{7}{2} & 4 \end{bmatrix}$

(C) $\begin{bmatrix} \frac{3}{2} & \frac{5}{2} & \frac{7}{2} \\ 2 & 3 & 4 \end{bmatrix}$

(D) $\begin{bmatrix} \frac{3}{2} & 2 & \frac{5}{2} \\ 3 & 3 & \frac{7}{2} \end{bmatrix}$

Answer –

(C) $\begin{bmatrix} \frac{3}{2} & \frac{5}{2} & \frac{7}{2} \\ 2 & 3 & 4 \end{bmatrix}$

Q17 - (a) The distance of the plane $\vec{r}.\left(\frac{2}{7}\hat{\imath}-\frac{3}{7}\hat{\jmath}+\frac{6}{7}\hat{k}\right)=4$ from the origin is

(A) 4

(B) 28

(C) $\frac{1}{7}$

(D) $\sqrt{7}$

Answer - (A) 4

OR

(b) If $\vec{a}$ and $\vec{b}$ are unit vectors, then the value of $\left(\vec{a}.\vec{b}\right)^2+\left|\vec{a}\times\vec{b}\right|^2$ is

(A) 0

(B) 1

(C) 2

(D) $sin\theta+cos\theta$

Answer - (B) 1

Q18 - The integrating factor of the following differential equation is $\frac{dy}{dx}+4\frac{y}{x}=\frac{1}{x}$

(A) $4x$

(B) x^2

(C) x^4

(D) $\frac{4}{x}$

Answer - (C) x^4

Q19 – (a) $\int\frac{cos^2x}{1+sinx}dx=$

(A) $x-cosx+c$

(B) $x+cosx+c$

(C) $x+sinx+c$

(D) $x-sinx+c$

Answer - (B) $x+cosx+c$

OR

(b) $\int \frac{(logx)^2}{x} dx =$

(A) $\left(\frac{logx}{3}\right)^3 + C$

(B) $\frac{logx^3}{3} + C$

(C) $\frac{(logx)^3}{3} + C$

(D) $\left(log\frac{x}{3}\right)^3 + C$

Answer - (C) $\frac{(logx)^3}{3} + C$

Q20 - Which one of the following statements is true?

(A) Every scalar matrix is an identity matrix.

(B) Every identity matrix is a scalar matrix.

(C) Every diagonal matrix is an identity matrix.

(D) A square matrix whose each element is 1 is an identity matrix.

Answer - (B) Every identity matrix is a scalar matrix.

Q21 - Fill in the blanks :

(a) Let $*$ be a binary operation defined by a*b = b – 2a, then the value of (1*2)*3 is ____.

(b) A relation R on any set A is said to be ____, if $(a, b) \in R, (b, c) \in R \Rightarrow (a, c) \in R$ for all $a, b, c \in A$.

Answer – (a) 3

(b) Transitive

Q22 - Match Column - I statement with the correct option of Column – II.

Column-I	Column-II
(a) $\int sinx\, dx$	**(P)** $cos\, x + c$
(b) $\int sec^2x\, dx$	**(Q)** $log(sin\, x) + c$
	(R) $tan\, x + c$
	(S) $-cos\, x + c$

Answer –

(a) $\int sinx\, dx \quad \rightarrow$ (S) $-cos\, x + c$

(b) $\int sec^2x\, dx \rightarrow$ (R) $tan\, x + c$

Q23 - Fill in the blanks (attempt any two sub-parts from (a) to (d)) :

(a) Let f : R → R be defined by f(x) = 2^x, then $(f \circ f)(0)$ is equal to ____.

Answer – 2

(b) The value of $cos^{-1}\left(\frac{1}{2}\right) + 2sin^{-1}\left(\frac{1}{2}\right)$ is ____.

Answer – $\frac{2\pi}{3}$

(c) If $cot^{-1}\left(\frac{3}{4}\right) = x$, then the value of $cos\, x$ is ____.

Answer – $\frac{3}{4}$

(d) A binary operation $*$ on a set A is a function from ____ to A.

Answer – $A \times A$

Q24 - Answer any two sub-parts from (a) to (d) :

(a) Write the negation of the statement, "$\sqrt{5}$ is not a rational number".

Answer – $\sqrt{5}$ is a rational number

(b) Write the converse of the statement, "If x is a prime number, then x is an even number'.

Answer – If x is an even number, then x is a prime number.

(c) Write the contrapositive of the statement, "If a number is odd, then its square is not negative".

Answer – If the square of a number is negative, then the number is not odd.

(d) Combine the statements P and Q using 'if and only if' :

P : If two angles of a triangle are equal, then it is an isosceles triangle.

Q : If a triangle is an isosceles triangle, then its two angles are equal.

Answer – A triangle is isosceles if and only if its two angles are equal.

Q25 - Fill in the blanks (attempt any four sub-parts from the (a) to (f)) :

(a) The unit vector in the direction of the vector $\vec{a} = 2\hat{\imath} + \hat{\jmath} + 2\hat{k}$ is _____.

Answer – $\frac{1}{3}(2\hat{\imath} + \hat{\jmath} + 2\hat{k})$.

(b) The three coordinate planes divide the whole space into eight parts called _____.

Answer – Octants.

(c) $(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b}) =$ _____.

Answer – $2(\vec{a} \times \vec{b})$.

(d) The vector equation of a plane passing through the point (2, –3, 4) and perpendicular to the line with direction ratios 1, 3, –2 is _____.

Answer – $x + 3y - 2z + 15 = 0$.

(e) The intercept of the plane $2x - 3y + 5z - 30 = 0$ on the z-axis is _____.

Answer – 6.

(f) The Cartesian form of the equation of the line $\vec{r} = (3\hat{\imath} + 4\hat{\jmath} - 2\hat{k}) + \lambda(\hat{\imath} - \hat{\jmath} - 3\hat{k})$ is _____.

Answer – $\frac{x-3}{1} = \frac{y-4}{-1} = \frac{z+2}{-3}$

Q26 - For the ellipse $16x^2 + y^2 = 16$, find the following (attempt any four sub-parts from (a) to (f)) :

(a) Length of the major axis

(b) Length of the minor axis

(c) Eccentricity

(d) Foci

(e) Equation of directrices

(f) Length of the latus rectum

Answer -

(a) 8

(b) 2

(c) $\frac{\sqrt{15}}{4}$

(d) $(0, \pm\sqrt{15})$

(e) $y = \pm\frac{16}{\sqrt{(15)}}$

(f) ½

Q27 - Fill in the blanks (attempt any four sub-parts from (a) to (f)) :

(a) If $A = \begin{bmatrix} 2 & -1 & 5 \\ 3 & 8 & 7 \end{bmatrix}$, then A′ = _____.

Answer – $\begin{bmatrix} 2 & 3 \\ -1 & 8 \\ 5 & 7 \end{bmatrix}$

(b) If A is a skew-symmetric matrix, then all its diagonal elements are _____.

Answer – Zero.

(c) If $A = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$ and $B = [3 \quad -1 \quad 2]$, then AB = _____.

Answer – $\begin{bmatrix} -3 & 1 & -2 \\ 6 & -2 & 4 \\ 9 & -3 & 6 \end{bmatrix}$

(d) If $\begin{vmatrix} 2x & 1 \\ 2x-1 & 2x+1 \end{vmatrix} = 5$ and x > 0, then x = _____.

Answer – 1

(e) The points $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ are collinear, if $\begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ 1 & 1 & 1 \end{vmatrix} =$_____.

Answer – 0

(f) Let AX = B be a system of linear equations having unique solution, then the solution is given by X = _____.

Answer – $A^{-1}B$

Q28 - Evaluate the following integrals (attempt any four sub-parts from (i) to (vi)) :

(a) $\int \frac{cos\sqrt{x}}{\sqrt{x}} dx$

(b) $\int e^{3logx}.x^4 dx$

(c) $\int \frac{sec^2x}{\sqrt{1-tan^{2x}}} dx$

(d) $\int_0^{\frac{\pi}{2}} \frac{cos^3x}{sin^3x+cos^3x}$

(e) $\int cosecx(cosecx + cotx)dx$

(f) $\int_0^{\frac{\pi}{4}} sin2xdx$

Answer – (a) $2sin\sqrt{x} + C$

(b) $\frac{x^8}{8} + C$

(c) $\sin^{-1}(tanx) + C$

(d) $\frac{\pi}{4}$

(e) $-cotx - cosecx + C$

(f) $\frac{1}{2}$

Q29 - Fill in the blanks (attempt any six sub-parts from (a) to (i)) :

(a) $\lim_{x \to 1} \frac{x^4-1}{x-1} =$ _____.

(b) $\lim_{x \to 0} \frac{e^{2x}-1}{x}$ = _____.

(c) If $f(x) = \begin{cases} ax - 1, x \neq 3 \\ 5 \quad , \quad x = 3 \end{cases}$ is continuous at x = 3, then the value of a is _____.

(d) If $y = (x^2 - 1)^2, then \frac{d^2y}{dx^2} =$ _____.

(e) If $y = tan\sqrt{x}, then \frac{dy}{dx} =$ _____.

(f) If $f(x) = \frac{x(x^2-1)}{x+4}, then\ f'(1) =$ _____.

(g) The rate of change of area of a circle with respect to its radius r, when r = 4 cm, is _____.

(h) $f(x) = cosx, x \in (0, 2\pi)$ is increasing in the interval _____.

(i) The slope of the tangent to the curve $y = x^3 - x\ at\ x = 2$ is _____.

Answer – (a) 4

(b) 2

(c) 2

(d) $12x^2 - 4$

(e) $\frac{\sec^2(\sqrt{x})}{2\sqrt{x}}$

(f) $\frac{2}{5}$

(g) 8π

(h) $(\pi, 2\pi)$

(i) 11

SECTION - B

Q30 - Using determinants, determine whether the points (a, b + c), (b, c + a) and (c, a + b) form a triangle or not.

Answer –

To determine whether the points

$$A(a,\, b+c), \quad B(b,\, c+a), \quad C(c,\, a+b)$$

form a triangle, we check whether they are collinear using the determinant condition.

The points will form a triangle if

$$\begin{vmatrix} a & b+c & 1 \\ b & c+a & 1 \\ c & a+b & 1 \end{vmatrix} \neq 0$$

Now evaluate the determinant.

$$\Delta = \begin{vmatrix} a & b+c & 1 \\ b & c+a & 1 \\ c & a+b & 1 \end{vmatrix}$$

Apply the operation:

$$R_1 \to R_1 - R_2, \quad R_2 \to R_2 - R_3$$

Then,

$$= \begin{vmatrix} a-b & (b+c)-(c+a) & 0 \\ b-c & (c+a)-(a+b) & 0 \\ c & a+b & 1 \end{vmatrix}$$

Simplify:

$$= \begin{vmatrix} a-b & b-a & 0 \\ b-c & c-b & 0 \\ c & a+b & 1 \end{vmatrix}$$

Factor:

$$b-a = -(a-b), \quad c-b = -(b-c)$$

So first two rows become:

$$= \begin{vmatrix} a-b & -(a-b) & 0 \\ b-c & -(b-c) & 0 \\ c & a+b & 1 \end{vmatrix}$$

Take common factors:

$$= (a-b)(b-c) \begin{vmatrix} 1 & -1 & 0 \\ 1 & -1 & 0 \\ c & a+b & 1 \end{vmatrix}$$

Now first two rows are identical.

Therefore determinant:

$$\Delta = 0$$

Hence, the three points are collinear.

Final conclusion:

$$\boxed{\text{The points do not form a triangle (they are collinear).}}$$

OR

Prove that $\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} = (5x+4)(4-x)^2$

Answer –

We have to prove

$$\Delta = \begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix}$$

Step 1: Apply row operations

Let

$$R_2 \to R_2 - R_1, \qquad R_3 \to R_3 - R_1$$

Then,

$$\Delta = \begin{vmatrix} x+4 & 2x & 2x \\ x-4 & 4-x & 0 \\ x-4 & 0 & 4-x \end{vmatrix}$$

Step 2: Factor

Notice:

$$x - 4 = -(4 - x)$$

So take $(4 - x)$ common from R_2 and R_3:

$$\Delta = (4-x)^2 \begin{vmatrix} x+4 & 2x & 2x \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix}$$

Step 3: Expand along first row

$$= (4-x)^2 \left[(x+4) \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} - 2x \begin{vmatrix} -1 & 0 \\ -1 & 1 \end{vmatrix} + 2x \begin{vmatrix} -1 & 1 \\ -1 & 0 \end{vmatrix} \right]$$

Now compute minors:

$$\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$$

$$\begin{vmatrix} -1 & 0 \\ -1 & 1 \end{vmatrix} = -1$$

$$\begin{vmatrix} -1 & 1 \\ -1 & 0 \end{vmatrix} = 1$$

Substitute:

$$= (4-x)^2 [(x+4) - 2x(-1) + 2x(1)]$$

$$= (4-x)^2 [(x+4) + 2x + 2x]$$

$$= (4-x)^2 (x+4)$$

Final Result

$$\boxed{\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} = (5x+4)(4-x)^2}$$

Hence proved.

Q31 - Solve for x : $2tan^{-1}(cos\ x) = tan^{-1}(2cosec\ x)$

Answer –

Given equation:

$$2\tan^{-1}(\cos x) = \tan^{-1}(2\,\text{cosec}\ x)$$

Step 1: Use the identity

$$2\tan^{-1}a = \tan^{-1}\left(\frac{2a}{1-a^2}\right)$$

Let $a = \cos x$.

$$2\tan^{-1}(\cos x) = \tan^{-1}\left(\frac{2\cos x}{1-\cos^2 x}\right)$$

Since

$$1-\cos^2 x = \sin^2 x,$$

$$= \tan^{-1}\left(\frac{2\cos x}{\sin^2 x}\right)$$

Step 2: Equate arguments

$$\tan^{-1}\left(\frac{2\cos x}{\sin^2 x}\right) = \tan^{-1}(2\operatorname{cosec} x)$$

So,

$$\frac{2\cos x}{\sin^2 x} = \frac{2}{\sin x}$$

Step 3: Simplify

Cancel 2:

$$\frac{\cos x}{\sin^2 x} = \frac{1}{\sin x}$$

Multiply by $\sin^2 x$:

$$\cos x = \sin x$$

Step 4: Solve

$$\tan x = 1$$

$$x = \frac{\pi}{4} + n\pi, \quad n \in \mathbb{Z}$$

OR

If $f: \{3,4,5,6\} \rightarrow \{4,5,6,10\}$ and $g: \{4,5,6,10\} \rightarrow \{8,12,16\}$ are two functions defined by $f(3) = 4, f(4) = 5, f95) = f(6) = 6$ and $g(4) = g(5) = 8, g(6) = g(10) = 12$, then find $g \circ f$.

Answer –

Given:

$$f : \{3,4,5,6\} \to \{4,5,6,10\}$$

$$g : \{4,5,6,10\} \to \{8,12,16\}$$

Defined by:

$$f(3) = 4, \quad f(4) = 5, \quad f(5) = 6, \quad f(6) = 6$$

$$g(4) = 8, \quad g(5) = 8, \quad g(6) = 12, \quad g(10) = 12$$

Find $g \circ f$

By definition,

$$(g \circ f)(x) = g(f(x))$$

Compute for each element of the domain of f:

$$(g \circ f)(3) = g(f(3)) = g(4) = 8$$

$$(g \circ f)(4) = g(f(4)) = g(5) = 8$$

$$(g \circ f)(5) = g(f(5)) = g(6) = 12$$

$$(g \circ f)(6) = g(f(6)) = g(6) = 12$$

Final Answer

$$g \circ f = \{(3,8), (4,8), (5,12), (6,12)\}$$

or equivalently,

$$(g \circ f)(3) = 8, \quad (g \circ f)(4) = 8, \quad (g \circ f)(5) = 12, \quad (g \circ f)(6) = 12$$

Q32 - Find the angle between the x-axis and the line joining the points (5, 4) and (6, 3).

Answer –

Given points:

$$A(5,4), \quad B(6,3)$$

Step 1: Find slope of the line

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{3-4}{6-5}$$

$$m = \frac{-1}{1} = -1$$

Step 2: Angle with the x-axis

If a line makes an angle θ with the positive direction of the x-axis, then

$$\tan\theta = m$$

$$\tan\theta = -1$$

The principal angle whose tangent is -1 is

$$\theta = -\frac{\pi}{4}$$

But the angle between a line and the x-axis is taken as an acute angle, so

$$\boxed{\theta = \frac{\pi}{4}}$$

Q33 – Solve the following differential equation : $x\frac{dy}{dx} + 2y = x^2$.

Answer –

Given differential equation:

$$x\frac{dy}{dx} + 2y = x^2$$

Step 1: Write in standard linear form

Divide by x (where $x \neq 0$):

$$\frac{dy}{dx} + \frac{2}{x}y = x$$

This is of the form

$$\frac{dy}{dx} + P(x)y = Q(x)$$

where

$$P(x) = \frac{2}{x}, \quad Q(x) = x$$

Step 2: Find the Integrating Factor (I.F.)

$$\text{I.F.} = e^{\int P(x)\,dx} = e^{\int \frac{2}{x}\,dx}$$

$$= e^{2\ln|x|} = x^2$$

Step 3: Multiply the equation by x^2

$$x^2\frac{dy}{dx} + 2xy = x^3$$

Left-hand side becomes:

$$\frac{d}{dx}(x^2y)$$

So,

$$\frac{d}{dx}(x^2y) = x^3$$

Step 4: Integrate

$$x^2y = \int x^3\,dx$$

$$= \frac{x^4}{4} + C$$

Step 5: Solve for y

$$y = \frac{x^4}{4x^2} + \frac{C}{x^2}$$

$$\boxed{y = \frac{x^2}{4} + \frac{C}{x^2}}$$

Q34 – A vector makes angles $\frac{\pi}{3}$ *and* $\frac{\pi}{2}$ with $\overrightarrow{OX}$ and $\overrightarrow{OY}$ respectively. Find the angle made by it with $\overrightarrow{OZ}$.

Answer –

Let the vector make angles

$$\alpha = \frac{\pi}{3} \quad \text{with } OX,$$

$$\beta = \frac{\pi}{2} \quad \text{with } OY,$$

$$\gamma \quad \text{with } OZ.$$

For a vector in 3D, the direction cosines satisfy:

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

Now compute:

$$\cos\alpha = \cos\frac{\pi}{3} = \frac{1}{2}$$
$$\cos\beta = \cos\frac{\pi}{2} = 0$$

Substitute:

$$\left(\frac{1}{2}\right)^2 + 0^2 + \cos^2\gamma = 1$$
$$\frac{1}{4} + \cos^2\gamma = 1$$
$$\cos^2\gamma = \frac{3}{4}$$
$$\cos\gamma = \pm\frac{\sqrt{3}}{2}$$

Taking the principal angle (between 0 and π):

$$\boxed{\gamma = \frac{\pi}{6}}$$

Q35 – Find A^{-1} for $A = \begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix}$.

Answer –

Given

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix}$$

For a 2×2 matrix

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$
$$|A| = ad - bc$$

and

$$A^{-1} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Step 1: Compute determinant

Here,

$$a = 1, \quad b = 2, \quad c = 2, \quad d = 5$$

$$|A| = ad - bc$$

$$= (1)(5) - (2)(2)$$

$$= 5 - 4 = 1$$

Step 2: Find inverse

$$A^{-1} = \frac{1}{1}\begin{bmatrix} 5 & -2 \\ -2 & 1 \end{bmatrix}$$

$$\boxed{A^{-1} = \begin{bmatrix} 5 & -2 \\ -2 & 1 \end{bmatrix}}$$

Q36 – Find the ratio in which the line segment joining (2, –3) and (5, 6) is divided by y-axis.

Answer –

Let the points be

$$A(2, -3), \quad B(5, 6)$$

Suppose the y-axis divides AB at point P in the ratio $m : n$ (internally).

Using section formula:

$$x_P = \frac{mx_2 + nx_1}{m + n}$$

Since P lies on the y-axis,

$$x_P = 0$$

Substitute $x_1 = 2, \ x_2 = 5$:

$$0 = \frac{5m + 2n}{m + n}$$

So,

$$5m + 2n = 0$$

$$5m = -2n$$

$$\frac{m}{n} = -\frac{2}{5}$$

Negative sign shows the division is external.

Hence the ratio is

$$\boxed{2 : 5 \text{ (externally)}}$$

Q37 - Using differentials, find the approximate value of $\sqrt{0.6}$.

Answer -

Let

$$y = \sqrt{x}$$

We approximate $\sqrt{0.6}$ using differentials near $x = 0.64$ (since $0.64 = 0.8^2$).

Take

$$x_0 = 0.64$$

$$y_0 = \sqrt{0.64} = 0.8$$

Now,

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$$

At $x = 0.64$:

$$\frac{dy}{dx} = \frac{1}{2(0.8)} = \frac{1}{1.6} = 0.625$$

Now,

$$dx = 0.6 - 0.64 = -0.04$$

Using differential:

$$dy = \frac{dy}{dx}\, dx$$

$$dy = 0.625(-0.04)$$

$$dy = -0.025$$

So,

$$\sqrt{0.6} \approx y_0 + dy$$

$$= 0.8 - 0.025$$

$$= 0.775$$

$$\boxed{\sqrt{0.6} \approx 0.775}$$

OR

Find $\frac{dy}{dx}$ for $y = x^{x\cos x}$.

Answer –

Given

$$y = x^{x\cos x}$$

Step 1: Take logarithm

$$\ln y = x\cos x\,\ln x$$

Step 2: Differentiate implicitly

$$\frac{1}{y}\frac{dy}{dx} = \frac{d}{dx}(x\cos x\,\ln x)$$

Use product rule for three factors.

Let $u = x\cos x$, $v = \ln x$.

$$\frac{d}{dx}(uv) = u'v + uv'$$

First find u':

$$u = x\cos x$$

$$u' = \cos x - x\sin x$$

Now differentiate:

$$\frac{1}{y}\frac{dy}{dx} = (\cos x - x\sin x)\ln x + x\cos x \cdot \frac{1}{x}$$

$$= (\cos x - x\sin x)\ln x + \cos x$$

Step 3: Multiply by y

$$\frac{dy}{dx} = y\,[(\cos x - x\sin x)\ln x + \cos x]$$

Since $y = x^{x\cos x}$,

$$\boxed{\frac{dy}{dx} = x^{x\cos x}\,[(\cos x - x\sin x)\ln x + \cos x]}$$

Q38 - Find the vector equation of a line passing through the point (4, 3, 0) and parallel to the line joining the points (2, 5, –1) and (–1, 4, 3).

Answer –

Given points

$$A(2,5,-1), \quad B(-1,4,3)$$

Step 1: Find direction vector of line AB

$$\vec{AB} = B - A$$

$$= (-1-2,\ 4-5,\ 3-(-1))$$

$$= (-3,\ -1,\ 4)$$

So a direction vector parallel to the required line is

$$\vec{d} = -3\mathbf{i} - \mathbf{j} + 4\mathbf{k}$$

Step 2: Write vector equation

Line passes through point

$$(4,3,0)$$

Position vector of the point:

$$\vec{r}_0 = 4\mathbf{i} + 3\mathbf{j}$$

Vector equation of a line:

$$\vec{r} = \vec{r}_0 + \lambda\vec{d}$$

Hence,

$$\boxed{\vec{r} = (4\mathbf{i} + 3\mathbf{j}) + \lambda(-3\mathbf{i} - \mathbf{j} + 4\mathbf{k})}$$

OR

Find a vector of magnitude 6 which is perpendicular to both the vectors $\vec{a} = 4\hat{\imath} - \hat{\jmath} + 3\hat{k}$ and $\vec{b} = -2\hat{\imath} + \hat{\jmath} - 2\hat{k}$.

Answer -

Given

$$\vec{a} = 4\hat{i} - \hat{j} + 3\hat{k}, \quad \vec{b} = -2\hat{i} + \hat{j} - 2\hat{k}$$

A vector perpendicular to both $\vec{a}$ and $\vec{b}$ is

$$\vec{a} \times \vec{b}$$

Step 1: Compute cross product

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -1 & 3 \\ -2 & 1 & -2 \end{vmatrix}$$

$$= \hat{i}\,[(-1)(-2) - (3)(1)] - \hat{j}\,[(4)(-2) - (3)(-2)] + \hat{k}\,[(4)(1) - (-1)(-2)]$$

$$= \hat{i}(2-3) - \hat{j}(-8+6) + \hat{k}(4-2)$$

$$= -\hat{i} + 2\hat{j} + 2\hat{k}$$

Step 2: Find magnitude

$$|\vec{a} \times \vec{b}| = \sqrt{(-1)^2 + 2^2 + 2^2} = \sqrt{9} = 3$$

Step 3: Make magnitude 6

Unit vector in this direction:

$$\frac{-\hat{i} + 2\hat{j} + 2\hat{k}}{3}$$

Multiply by 6:

$$6 \cdot \frac{-\hat{i} + 2\hat{j} + 2\hat{k}}{3} = 2(-\hat{i} + 2\hat{j} + 2\hat{k})$$

$$= -2\hat{i} + 4\hat{j} + 4\hat{k}$$

Final Answer

$$\boxed{-2\hat{i} + 4\hat{j} + 4\hat{k}}$$

Q39 - Find : $\int \frac{x^2}{x^2+6x+12} dx$

Answer –

$$I = \int \frac{x^2}{x^2 + 6x + 12}\, dx$$

Step 1: Rewrite numerator

$$x^2 = (x^2 + 6x + 12) - (6x + 12)$$

So,

$$I = \int \left[1 - \frac{6x + 12}{x^2 + 6x + 12}\right] dx$$

$$= \int 1\,dx - \int \frac{6x+12}{x^2+6x+12}\,dx$$

$$= x - \int \frac{6x+12}{x^2+6x+12}\,dx$$

Step 2: Split numerator

$$6x+12 = 3(2x+6) - 6$$

$$I = x - 3\int \frac{2x+6}{x^2+6x+12}\,dx + 6\int \frac{dx}{x^2+6x+12}$$

Step 3: First integral

Let $u = x^2+6x+12$.

$$\int \frac{2x+6}{x^2+6x+12}dx = \ln|x^2+6x+12|$$

So,

$$-3\ln|x^2+6x+12|$$

Step 4: Second integral

Complete the square:

$$x^2+6x+12 = (x+3)^2+3$$

$$\int \frac{dx}{(x+3)^2+3} = \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x+3}{\sqrt{3}}\right)$$

So,

$$6\cdot\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x+3}{\sqrt{3}}\right) = 2\sqrt{3}\tan^{-1}\left(\frac{x+3}{\sqrt{3}}\right)$$

Final Answer

$$\boxed{x - 3\ln|x^2+6x+12| + 2\sqrt{3}\,\tan^{-1}\left(\frac{x+3}{\sqrt{3}}\right) + C}$$

OR

Evaluate : $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{1+\sqrt{tanx}}\,dx$

Answer –

$$I = \int_{\pi/6}^{\pi/3} \frac{1}{1+\sqrt{\tan x}}\,dx$$

Use the property:

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

Here,

$$a = \frac{\pi}{6}, \quad b = \frac{\pi}{3}$$

So,

$$a + b = \frac{\pi}{2}$$

Thus,

$$I = \int_{\pi/6}^{\pi/3} \frac{1}{1+\sqrt{\tan(\frac{\pi}{2}-x)}}\,dx$$

Since

$$\tan\left(\frac{\pi}{2}-x\right) = \cot x = \frac{1}{\tan x}$$

$$\sqrt{\tan\left(\frac{\pi}{2}-x\right)} = \frac{1}{\sqrt{\tan x}}$$

So,

$$I = \int_{\pi/6}^{\pi/3} \frac{1}{1+\frac{1}{\sqrt{\tan x}}}\,dx$$

$$= \int_{\pi/6}^{\pi/3} \frac{\sqrt{\tan x}}{1+\sqrt{\tan x}}\,dx$$

Now add the two forms:

$$2I = \int_{\pi/6}^{\pi/3} \left[\frac{1}{1+\sqrt{\tan x}} + \frac{\sqrt{\tan x}}{1+\sqrt{\tan x}}\right] dx$$

$$= \int_{\pi/6}^{\pi/3} 1\,dx$$

$$= \frac{\pi}{3} - \frac{\pi}{6}$$

$$= \frac{\pi}{6}$$

$$\boxed{I = \frac{\pi}{12}}$$

Q40 - If $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$ **and** $B = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$ **then find** $A^2 + B^2 + 2AB$.

Answer –

Given

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$$

We use the identity:

$$A^2 + B^2 + 2AB = (A+B)^2$$

Step 1: Find $A + B$

$$A + B = \begin{bmatrix} 1+2 & 0+0 & 2+1 \\ 0+2 & 2+1 & 1+3 \\ 2+1 & 0-1 & 3+0 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 3 \\ 2 & 3 & 4 \\ 3 & -1 & 3 \end{bmatrix}$$

Step 2: Multiply $(A+B)(A+B)$

$$(A+B)^2 = \begin{bmatrix} 3 & 0 & 3 \\ 2 & 3 & 4 \\ 3 & -1 & 3 \end{bmatrix} \begin{bmatrix} 3 & 0 & 3 \\ 2 & 3 & 4 \\ 3 & -1 & 3 \end{bmatrix}$$

Now multiply row by column.

First row calculations

$$(1,1) = 3 \cdot 3 + 0 \cdot 2 + 3 \cdot 3 = 9 + 0 + 9 = 18$$

$$(1,2) = 3 \cdot 0 + 0 \cdot 3 + 3 \cdot (-1) = 0 + 0 - 3 = -3$$

$$(1,3) = 3 \cdot 3 + 0 \cdot 4 + 3 \cdot 3 = 9 + 0 + 9 = 18$$

Second row calculations

$$(2,1) = 2 \cdot 3 + 3 \cdot 2 + 4 \cdot 3 = 6 + 6 + 12 = 24$$

$$(2,2) = 2 \cdot 0 + 3 \cdot 3 + 4 \cdot (-1) = 0 + 9 - 4 = 5$$

$$(2,3) = 2 \cdot 3 + 3 \cdot 4 + 4 \cdot 3 = 6 + 12 + 12 = 30$$

Third row calculations

$$(3,1) = 3 \cdot 3 + (-1) \cdot 2 + 3 \cdot 3 = 9 - 2 + 9 = 16$$

$$(3,2) = 3 \cdot 0 + (-1) \cdot 3 + 3 \cdot (-1) = 0 - 3 - 3 = -6$$

$$(3,3) = 3 \cdot 3 + (-1) \cdot 4 + 3 \cdot 3 = 9 - 4 + 9 = 14$$

Final Answer

$$A^2 + B^2 + 2AB = \boxed{\begin{bmatrix} 18 & -3 & 18 \\ 24 & 5 & 30 \\ 16 & -6 & 14 \end{bmatrix}}$$

Q41 - The vertices of a ΔABC are A (– 3, 3), B (5, –2) and C (–1, –4). If M and N are the midpoints of AB and AC respectively, then show that $MN = \frac{1}{2}BC$.

Answer –

Given vertices:

$$A(-3,3), \quad B(5,-2), \quad C(-1,-4)$$

Let M and N be the midpoints of AB and AC respectively.

Step 1: Find midpoint M of AB

$$M = \left(\frac{-3+5}{2}, \frac{3+(-2)}{2}\right)$$

$$= \left(\frac{2}{2}, \frac{1}{2}\right)$$

$$= (1, \tfrac{1}{2})$$

Step 2: Find midpoint N of AC

$$N = \left(\frac{-3+(-1)}{2}, \frac{3+(-4)}{2}\right)$$

$$= \left(\frac{-4}{2}, \frac{-1}{2}\right)$$

$$= (-2, -\tfrac{1}{2})$$

Step 3: Find vector MN

$$\vec{MN} = N - M$$

$$= (-2-1,\ -\tfrac{1}{2} - \tfrac{1}{2})$$

$$= (-3, -1)$$

Step 4: Find vector BC

$$\vec{BC} = C - B$$

$$= (-1-5,\ -4-(-2))$$

$$= (-6, -2)$$

Step 5: Compare

$$\frac{1}{2}\vec{BC} = \frac{1}{2}(-6, -2) = (-3, -1)$$

$$\vec{MN} = (-3, -1) = \frac{1}{2}\vec{BC}$$

Final Result

$$\boxed{\vec{MN} = \frac{1}{2}\vec{BC}}$$

Hence proved.

OR

Find the equation of the circle which passes through the points (2, 3) and (–1, 1), and whose centre lies on the line $x - 3y - 11 = 0$.

Answer –

Let the centre of the circle be $C(h, k)$.

Since the circle passes through

$$A(2,3) \quad \text{and} \quad B(-1,1),$$

the centre must be equidistant from A and B.

Step 1: Use equal distance condition

$$CA = CB$$

$$(h-2)^2 + (k-3)^2 = (h+1)^2 + (k-1)^2$$

Expand:

$$h^2 - 4h + 4 + k^2 - 6k + 9 = h^2 + 2h + 1 + k^2 - 2k + 1$$

Cancel h^2 and k^2:

$$-4h - 6k + 13 = 2h - 2k + 2$$

$$-6h - 4k + 11 = 0$$

Divide by -1:

$$6h + 4k - 11 = 0$$

Step 2: Centre lies on line

Given:

$$h - 3k - 11 = 0$$

Step 3: Solve the two equations

$$6h + 4k - 11 = 0$$

$$h - 3k - 11 = 0$$

From second:

$$h = 3k + 11$$

Substitute:

$$6(3k + 11) + 4k - 11 = 0$$

$$18k + 66 + 4k - 11 = 0$$

$$22k + 55 = 0$$

$$k = -\frac{55}{22} = -\frac{5}{2}$$

$$h = 3\left(-\frac{5}{2}\right) + 11$$

$$= -\frac{15}{2} + \frac{22}{2}$$

$$= \frac{7}{2}$$

So centre:

$$C\left(\frac{7}{2}, -\frac{5}{2}\right)$$

Step 4: Find radius

$$r^2 = \left(\frac{7}{2} - 2\right)^2 + \left(-\frac{5}{2} - 3\right)^2$$

$$= \left(\frac{3}{2}\right)^2 + \left(-\frac{11}{2}\right)^2$$

$$= \frac{9}{4} + \frac{121}{4}$$

$$= \frac{130}{4} = \frac{65}{2}$$

Final Equation

$$\boxed{\left(x - \frac{7}{2}\right)^2 + \left(y + \frac{5}{2}\right)^2 = \frac{65}{2}}$$

Q42 – If $x = a\left(\cos\theta + \log\tan\frac{\theta}{2}\right)$ and $y = a\sin\theta$, then find $\frac{d^2y}{dx^2}$.

Answer –

Given parametric equations:

$$x = a\left(\cos\theta + \log\tan\frac{\theta}{2}\right), \qquad y = a\sin\theta$$

Step 1: Differentiate w.r.t. θ

First,

$$\frac{dy}{d\theta} = a\cos\theta$$

Now differentiate x:

$$\frac{dx}{d\theta} = a\left(-\sin\theta + \frac{d}{d\theta}\log\tan\frac{\theta}{2}\right)$$

Now,

$$\frac{d}{d\theta}\log\tan\frac{\theta}{2} = \frac{1}{\tan\frac{\theta}{2}}\cdot\frac{d}{d\theta}\tan\frac{\theta}{2}$$

$$= \frac{1}{\tan\frac{\theta}{2}}\cdot\frac{1}{2}\sec^2\frac{\theta}{2}$$

Using identity,

$$\frac{\sec^2\frac{\theta}{2}}{\tan\frac{\theta}{2}} = \frac{2}{\sin\theta}$$

Hence,

$$\frac{d}{d\theta}\log\tan\frac{\theta}{2} = \frac{1}{\sin\theta}$$

So,

$$\frac{dx}{d\theta} = a\left(-\sin\theta + \frac{1}{\sin\theta}\right)$$

$$= a\frac{1-\sin^2\theta}{\sin\theta}$$

$$= a\frac{\cos^2\theta}{\sin\theta}$$

Step 2: First derivative

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{a\cos\theta}{a\frac{\cos^2\theta}{\sin\theta}}$$

$$= \frac{\cos\theta}{\cos^2\theta/\sin\theta} = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

Step 3: Second derivative

$$\frac{d^2y}{dx^2} = \frac{d}{d\theta}(\tan\theta) \Big/ \frac{dx}{d\theta}$$

$$= \frac{\sec^2\theta}{a\frac{\cos^2\theta}{\sin\theta}}$$

$$= \frac{1/\cos^2\theta}{a\cos^2\theta/\sin\theta}$$

$$= \frac{\sin\theta}{a\cos^4\theta}$$

Final Answer

$$\boxed{\frac{d^2y}{dx^2} = \frac{\sin\theta}{a\cos^4\theta}}$$

Q43 - Let $f:\ R^+ \to (-4, \infty)$ be a function defined as $f(x) = x^2 - 4$, then show that f is invertible and find the inverse.

Answer –

Given

$$f : \mathbb{R}^+ \to (-4, \infty), \qquad f(x) = x^2 - 4$$

Step 1: Show that f is one–one

Let

$$f(x_1) = f(x_2)$$

$$x_1^2 - 4 = x_2^2 - 4$$

$$x_1^2 = x_2^2$$

$$x_1 = \pm x_2$$

But since domain is $\mathbb{R}^+$ (positive real numbers),

$$x_1 = x_2$$

Hence f is one–one.

Step 2: Show that f is onto

Let $y \in (-4, \infty)$.

$$y = x^2 - 4$$
$$x^2 = y + 4$$
$$x = \sqrt{y+4}$$

Since $y > -4$, $y + 4 > 0$, so $x \in \mathbb{R}^+$.

Thus for every $y \in (-4, \infty)$, there exists $x \in \mathbb{R}^+$.

Hence f is onto.

Step 3: Find the inverse

$$y = x^2 - 4$$
$$x = \sqrt{y+4}$$

So,

$$\boxed{f^{-1}(x) = \sqrt{x+4}}$$

Thus, f is bijective and hence invertible.

Q44 - Show that the lines $\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$ and $\frac{x-2}{1} = \frac{y-4}{4} = \frac{z-6}{7}$ are coplanar. Also, find the equation of the plane containing these lines.

Answer –

Given lines:

$$\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$$
$$\frac{x-2}{1} = \frac{y-4}{4} = \frac{z-6}{7}$$

Step 1: Write in parametric form

For first line L_1:

$$x = -1 + 3t, \quad y = -3 + 5t, \quad z = -5 + 7t$$

Point $A(-1, -3, -5)$

Direction vector

$$\vec{d}_1 = (3, 5, 7)$$

For second line L_2:

$$x = 2 + s, \quad y = 4 + 4s, \quad z = 6 + 7s$$

Point $B(2, 4, 6)$

Direction vector

$$\vec{d}_2 = (1, 4, 7)$$

Step 2: Check coplanarity

Vector joining A and B:

$$\vec{AB} = (3, 7, 11)$$

Find cross product:

$$\vec{d}_1 \times \vec{d}_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 5 & 7 \\ 1 & 4 & 7 \end{vmatrix}$$

$$= (7, -14, 7)$$

Now scalar triple product:

$$\vec{AB} \cdot (\vec{d}_1 \times \vec{d}_2) = (3, 7, 11) \cdot (7, -14, 7)$$

$$= 21 - 98 + 77 = 0$$

Hence lines are coplanar.

Step 3: Equation of plane

Normal vector

$$\vec{n} = (7, -14, 7)$$

Divide by 7:

$$\vec{n} = (1, -2, 1)$$

Using point $A(-1, -3, -5)$:

$$1(x + 1) - 2(y + 3) + 1(z + 5) = 0$$

$$x - 2y + z = 0$$

Final Answer

The lines are coplanar and the required plane is

$$\boxed{x - 2y + z = 0}$$

OR

Find the vector and Cartesian equation of the plane passing through A (–2, 3, –1), B (–3, 7, –4) and C (–1, 0, –1). Also, find its intercepts on the three coordinate axes and its distance from the origin.

Answer -

Given points

$$A(-2,3,-1), \quad B(-3,7,-4), \quad C(-1,0,-1)$$

Step 1: Find two direction vectors

$$\vec{AB} = B - A = (-3+2,\ 7-3,\ -4+1) = (-1,4,-3)$$

$$\vec{AC} = C - A = (-1+2,\ 0-3,\ -1+1) = (1,-3,0)$$

Step 2: Find normal vector

$$\vec{n} = \vec{AB} \times \vec{AC}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 4 & -3 \\ 1 & -3 & 0 \end{vmatrix}$$

$$= \hat{i}(4\cdot 0 - (-3)(-3)) - \hat{j}((-1)\cdot 0 - (-3)\cdot 1) + \hat{k}((-1)(-3) - 4\cdot 1)$$

$$= \hat{i}(0-9) - \hat{j}(0+3) + \hat{k}(3-4)$$

$$= -9\hat{i} - 3\hat{j} - \hat{k}$$

Taking positive form:

$$\vec{n} = (9,3,1)$$

Step 3: Vector equation

Using point $A(-2,3,-1)$:

$$\vec{r}\cdot(9\hat{i} + 3\hat{j} + \hat{k}) = (-2,3,-1)\cdot(9,3,1)$$

$$= -18 + 9 - 1 = -10$$

So vector equation:

$$\boxed{\vec{r}\cdot(9\hat{i} + 3\hat{j} + \hat{k}) = -10}$$

Step 4: Cartesian equation

$$9x + 3y + z = -10$$

Step 5: Intercepts

For x-intercept: $y = z = 0$

$$9x = -10 \Rightarrow x = -\frac{10}{9}$$

For y-intercept: $x = z = 0$

$$3y = -10 \Rightarrow y = -\frac{10}{3}$$

For z-intercept: $x = y = 0$

$$z = -10$$

Step 6: Distance from origin

$$\text{Distance} = \frac{|9(0) + 3(0) + 1(0) + 10|}{\sqrt{9^2 + 3^2 + 1^2}}$$

$$= \frac{10}{\sqrt{91}}$$

Final Answers

Vector equation:

$$\boxed{\vec{r} \cdot (9\hat{i} + 3\hat{j} + \hat{k}) = -10}$$

Cartesian equation:

$$\boxed{9x + 3y + z + 10 = 0}$$

Intercepts:

$$\left(-\frac{10}{9}, 0, 0\right), \left(0, -\frac{10}{3}, 0\right), (0, 0, -10)$$

Distance from origin:

$$\boxed{\frac{10}{\sqrt{91}}}$$

Q45 - A cooperative society of farmers has 50 hectares of land to grow two crops A and B. The profits from crops A and B per hectare are estimated as ₹10,500 and ₹9,000 respectively. To control weeds, a liquid herbicide has to be used for crops A and B at the rate of 20 litres and 10 litres per hectare respectively. Further, not more than 800 litres of herbicide should be used in order to protect fish and wildlife in the pond which collects drainage from this land. How

much land should be allocated to each crop so as to maximize the profit? Form an LPP and solve it graphically.

Answer -

Let

x = hectares allocated to Crop A
y = hectares allocated to Crop B

Formulation of LPP

Land constraint:

$$x + y \leq 50$$

Herbicide constraint:

$$20x + 10y \leq 800$$

$$2x + y \leq 80$$

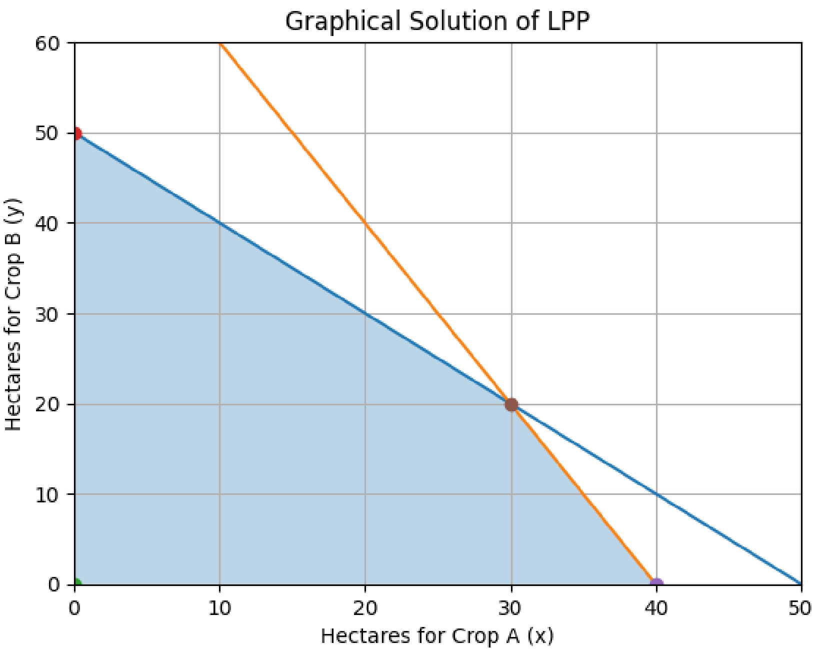
Non-negativity:

$$x \geq 0, \quad y \geq 0$$

Objective function (Profit):

$$Z = 10500x + 9000y$$

Maximise Z.



Corner Points (from graph)

$$(0,0), \quad (0,50), \quad (40,0), \quad (30,20)$$

Evaluate Z

$$Z(0,0) = 0$$

$$Z(0,50) = 9000(50) = 450000$$

$$Z(40,0) = 10500(40) = 420000$$

$$Z(30,20) = 10500(30) + 9000(20)$$

$$= 315000 + 180000$$

$$= 495000$$

Maximum Profit

$$Z_{\text{max}} = ₹4,95,000$$

Occurs at

$$\boxed{x = 30, \quad y = 20}$$

OR

Solve the following LPP by graphical method :

Minimize $Z = 20x + 10y$

subject to

$$x + 2y \le 40$$
$$3x + y \ge 30$$
$$4x + 3y \ge 60$$
$$x, y \ge 0$$

Answer –

Minimize

$$Z = 20x + 10y$$

Subject to

$$x + 2y \le 40$$

$$3x + 4y \le 30$$

$$4x + 3y \ge 60$$

$$x \ge 0, \quad y \ge 0$$

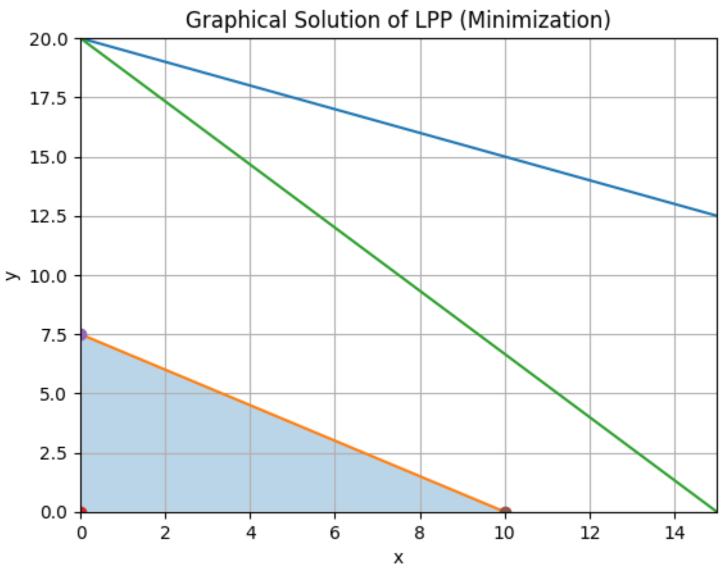
Step 1: Boundary lines (for graph)

$$x + 2y = 40 \Rightarrow (40, 0), (0, 20)$$

$$x + y = 10 \Rightarrow (10, 0), (0, 10)$$

$$4x + 5y = 30 \Rightarrow (7.5, 0), (0, 6)$$

Plot these lines and take the common region satisfying all inequalities (including $x, y \geq 0$).



Step 2: Feasible corner points (from graph)

$$(0, 0),\ (0, 6),\ (4, 2),\ (10, 0)$$

Step 3: Evaluate objective function

$$Z(0, 0) = 0$$

$$Z(0, 6) = 20(0) + 10(6) = 60$$

$$Z(4, 2) = 20(4) + 10(2) = 100$$

$$Z(10, 0) = 200$$

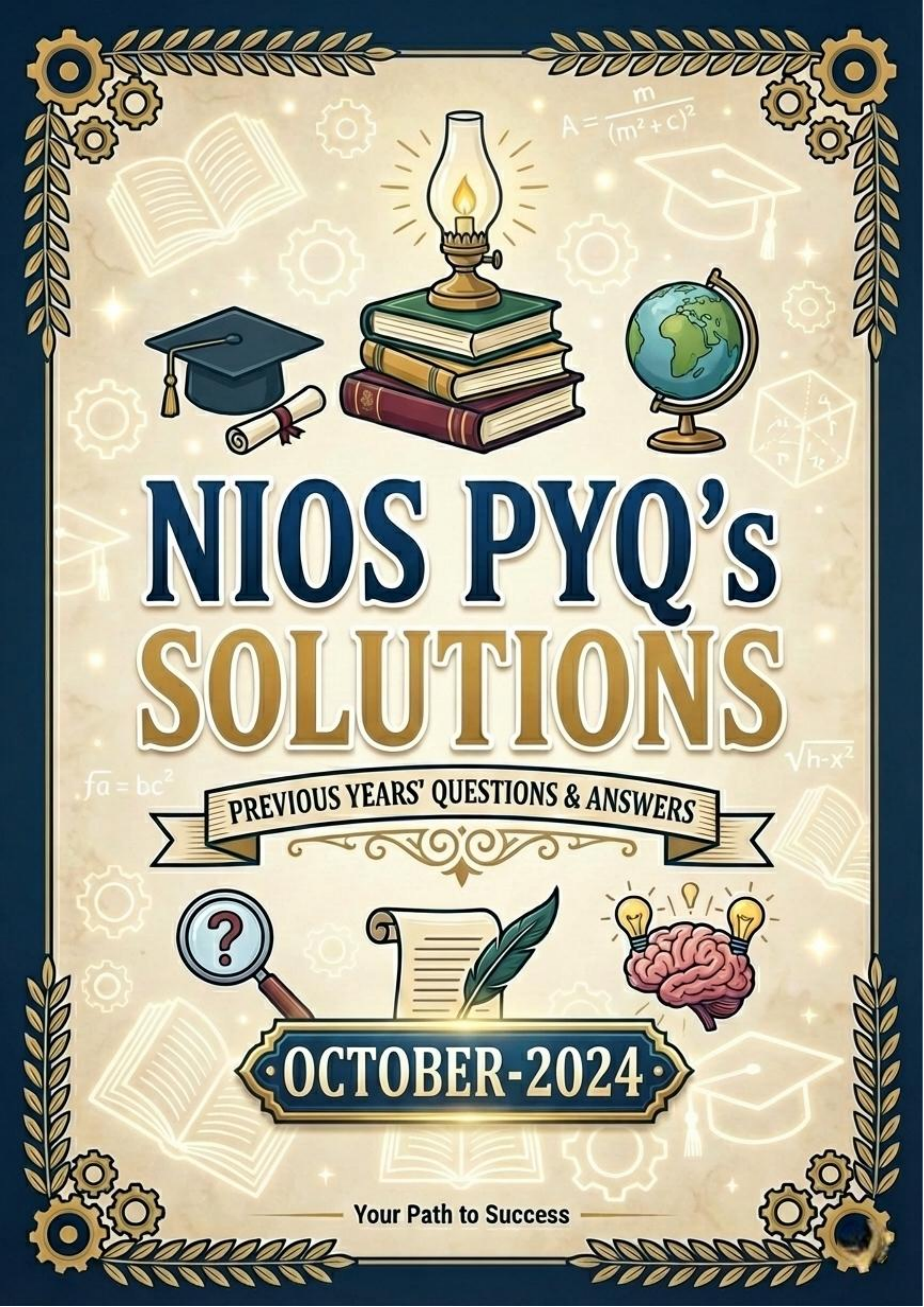
Final Answer

Minimum value occurs at

$$\boxed{(x, y) = (0, 0)}$$

$$\boxed{Z_{\text{min}} = 0}$$

Hence, the minimum value of the objective function is 0.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SECTION - A

Q1 - The points A (−1, −1), B (2, 3) and C (−2, 6) are the vertices of :

(A) an equilateral triangle

(B) an isosceles triangle

(C) a scalene triangle

(D) an isosceles right triangle

Answer - (D) an isosceles right triangle

Q2 – The slope of a line which makes an angle of 60° with the negative direction of x-axis, is :

(A) $\frac{1}{\sqrt{3}}$

(B) $-\frac{1}{\sqrt{3}}$

(C) $\sqrt{3}$

(D) $-\sqrt{3}$

Answer - (D) $-\sqrt{3}$

Q3 - For the hyperbola $\frac{x^2}{16} - \frac{y^2}{9}$, the length of the latus rectum is :

(A) 9/2 units

(B) 8/3 units

(C) 3 units

(D) 4 units

Answer - (A) 9/2 units

Q4 - The radius of the circle $4x^2 + 4y^2 - 2x + 3y - 6 = 0$ is:

(A) $\frac{\sqrt{37}}{4}$

(B) $\frac{\sqrt{109}}{8}$

(C) $\frac{\sqrt{71}}{6}$

(D) $\frac{\sqrt{107}}{8}$

Answer - (B) $\frac{\sqrt{109}}{8}$

Q5 - The equation $x + 7y - 4 = 0$ in the slope-intercept form is :

(A) $\frac{x}{4} + \frac{7y}{4} = 1$

(B) $x = -7y + 4$

(C) $y = -\frac{1}{7}x + \frac{4}{7}$

(D) $y = \frac{1}{7}x + \frac{4}{7}$

Answer - (C) $y = -\frac{1}{7}x + \frac{4}{7}$

Q6 - The two matrices $\begin{bmatrix} 2a & b \\ -4 & 6 \end{bmatrix}$ and $\begin{bmatrix} 3 & -2 \\ d & 3c \end{bmatrix}$ are equal for the values a, b, c, d :

(A) $a = \frac{3}{2}, b = -2, c = -2, d = 4$

(B) $a = -\frac{3}{2}, b = -2, c = 2, d = 4$

(C) $a = \frac{3}{2}, b = -2, c = 2, d = -4$

(D) $a = \frac{3}{2}, b = -2, c = 2, d = 4$

Answer - (C) $a = \frac{3}{2}, b = -2, c = 2, d = -4$

Q7 - For the matrix $A = \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 \\ -1 \\ 3 \end{bmatrix}$:

(A) AB exists

(B) BA exists

(C) AB and BA both exists

(D) Neither AB nor BA exists

Answer - (D) Neither AB nor BA exists

Q8 – If $A = \begin{bmatrix} 1 & 6 \\ 3 & 2 \end{bmatrix}$, then |A'|, where A' denotes the transpose of the matrix A, is :

(A) −16

(B) 16

(C) −8

(D) 8

Answer - (A) −16

Q9 - The relation R defined in the set A = {x ∈ I, 0 ≤ x ≤ 12} as R = {(a, b); a = b a, b ∈ A} is :

(A) reflexive

(B) transitive

(C) symmetrical

(D) an equivalence relation

Answer - (D) an equivalence relation

Q10 - The domain of the function y = $\sec^{-1}$ x is :

(A) [−1, 1]

(B) R

(C) x ≥ 1 or x ≤ −1

(D) R−{0}

Answer - (C) x ≥ 1 or x ≤ −1

Q11 - Let * be a binary operation on the set N of natural numbers defined by the rule a*b = ab, for all, a, b ∈ N, then :

(A) * is commutative.

(B) * is associative.

(C) * is both commutive and associative.

(D) * is neither commutive nor associative.

Answer - (C) * is both commutive and associative.

Q12 - The inverse of the function y = x^2, for all x ∈ R is :

(A) $f^{-1}(x) = -x$

(B) $f^{-1}(x) = x$

(C) $f^{-1}(x) = |x|$

(D) does not exist

Answer - (D) does not exist

Q13 - If $y = (1 - x^2)^5$, then $\frac{dy}{dx}$ is :

(A) $5(1 - x^2)^4$

(B) $(1 - 2x)^5$

(C) $5(1 - x^2)^4(-2x)$

(D) $5(1 - 2x)^4$

Answer - (C) $5(1 - x^2)^4(-2x)$

Q14 - If $y = sin2x\, cos3x$, then $\frac{dy}{dx}$ is :

(A) $2\, cos2x\, cos3x - 3\, sin2x\, sin3x$

(B) $2\, cos2x\, cos3x + 3\, sin2x\, sin3x$

(C) $6\, cos2x\, sin3x - 6\, cos2x\, sin3x$

(D) $6\, cos2x\, sin3x + 6\, cos2x\, sin3x$

Answer - (A) $2\, cos2x\, cos3x - 3\, sin2x\, sin3x$

Q15 - Which of the following is a vector quantity?

(A) Mass

(B) Density

(C) Force

(D) Time

Answer - (C) Force

Q16 - The unit vector parallel to the resultant of vector $\vec{a} = 3\hat{\imath} - 2\hat{\jmath} + \hat{k}$ and $\vec{b} = -2\hat{\imath} + 4\hat{\jmath} + \hat{k}$ is:

(A) $\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$

(B) $\frac{1}{5}(\hat{\imath} + 2\hat{\jmath} + 2\hat{k})$

(C) $\frac{1}{3}(\hat{\imath} + 2\hat{\jmath} + 2\hat{k})$

(D) $\pm\frac{1}{3}(\hat{\imath} + 2\hat{\jmath} + 2\hat{k})$

Answer - (C) $\frac{1}{3}(\hat{\imath} + 2\hat{\jmath} + 2\hat{k})$

Q17 - The distance of the point (3, 4, −5) from the plane $2x - 3y + 3z + 27 = 0$ is :

(A) $6\, units$

(B) $\sqrt{22} units$

(C) $\frac{6}{\sqrt{22}} units$

(D) $\sqrt{132} units$

Answer - (C) $\frac{6}{\sqrt{22}} units$

Q18 - The equation of the line passing through the points (1, 4, 7) and (3, −2, 5) is :

(A) $\frac{x-1}{2} = \frac{y-4}{-6} = \frac{z-7}{-2}$

(B) $\frac{x-1}{2} = \frac{y-4}{2} = \frac{z}{12}$

(C) $\frac{x-1}{2} = \frac{y-4}{6} = \frac{z-7}{-2}$

(D) $\frac{x-1}{2} = \frac{y-4}{-2} = \frac{z-7}{2}$

Answer - (A) $\frac{x-1}{2} = \frac{y-4}{-6} = \frac{z-7}{-2}$

Q19 - Converse of the statement "If x is divisible by 4, then x is even" is :

(A) If x is not divisible by 4, then x is not even.

(B) If x is even, then x is divisible by 4.

(C) If x is even, then x is not divisible by 4.

(D) If x is not divisible by 4, then x is even.

Answer - (B) If x is even, then x is divisible by 4.

Q20 - The degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 + x^6\left(\frac{dy}{dx}\right)^4 = 0$ is :

(A) 2 **(B) 4**

(C) 6 **(D) 10**

Answer - (A) 2

Q21 - Match Column - I statement with the correct option of Column – II.

Column-I	Column-II
(a) The cofactor of the element −2 in the matrix $\begin{bmatrix} -1 & 3 & 6 \\ 2 & 5 & -2 \\ 4 & 1 & 3 \end{bmatrix}$ **is**	**(P) 13**
(b) If $A = \begin{bmatrix} -4 & 5 \\ 2 & -3 \end{bmatrix}$**, then** $\lvert adjA \rvert$ **is**	**(Q) –2**
	(R) –13
	(S) 2

Answer –

(a) The cofactor of the element −2 in the matrix $\begin{bmatrix} -1 & 3 & 6 \\ 2 & 5 & -2 \\ 4 & 1 & 3 \end{bmatrix}$ is → (P) 13

(b) If $A = \begin{bmatrix} -4 & 5 \\ 2 & -3 \end{bmatrix}$, then $\lvert adjA \rvert$ is → (S) 2

Q22 - Fill in the blanks :

(a) If $y = \sqrt{sin^3 x}$, then $\frac{dy}{dx}$ is _________.

(b) The order of the differential equation $xdx + ydy = 0$ is _________.

Answer – (a) $\frac{dy}{dx} = \frac{3}{2}\sqrt{sinx}.cosx$

(b) 1 (one).

Q23 - Write True for correct statement and False for incorrect statements

(a) The relation "is a factor of" from R to R is reflexive and Transitive but not symmetric.

(b) The function f : R → R defined by f(x) = x^2 + 3 is one-one and onto

Answer – (a) True

(b) False

Q24 - Write the negation of each of the following statements :

(a) The number is less than 5.

(b) All prime numbers are odd

Answer – (a) The number is greater than or equal to 5.

(b) There exists a prime number which is not odd (i.e., even).

Q25 - Fill in the blanks :

(a) It is not possible to add two matrices of _________ orders.

(b) If A is an invertible matrix, then $(A^{-1})^{-1}$=_________.

(c) Three points are collinear if the area of the triangle formed by these three points is ________.

(d) A square matrix A is said to be a ________ matrix, if A′ = −A

Answer - (a) different

(b) A

(c) zero

(d) Skew-Symmetric

Q26 - Fill in the blanks :

(a) If $y = sin^{-1}(x^2)$, then $\frac{dy}{dx}$ is _________.

(b) $\int 3^{5x-3} dx$=_________.

(c) The general solution of the first order linear differential equation $\frac{dy}{dx} + Py = Q$ is _________.

(d) $\int_{-a}^{a} \frac{xe^{x^2}}{1+x^2} dx$ = _________.

Answer - (a) $-\frac{2x}{\sqrt{1-x^4}}$

(b) $\frac{3^{5x-3}}{5 \ln 3}$

(c) $ye^{\int (Pdx)} = \int (Qe^{\int (Pdx)} dx + C$

(d) 0

Q27 - Write True for correct statements and False for incorrect statements :

(a) $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx.$

(b) If $x = at^2$ and $y = 2at$, then = $\frac{d^2y}{dx^2} = t.$

(c) The slope of normal to curve $y = f(x)$ at (x_1, y_1) is given by $\left(\frac{dy}{dx}\right) at\ (x_1, y_1)$.

(d) The degree of a differential equation is the degree of the highest differential coefficient.

Answer –

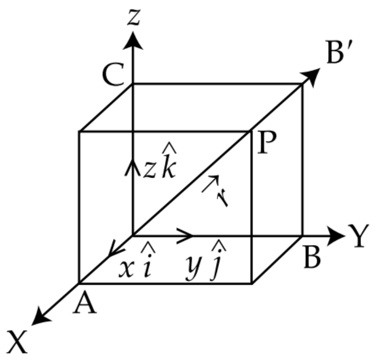
(a) True

(b) False

(c) False

(d) False

Q28 - Carefully study the figure given below and answer the following :



(a) What are $x\hat{\imath}, y\hat{\jmath}$ *and* $z\hat{k}$ called for the vector $\vec{r}$?

(b) If $|\overrightarrow{OA}| = 4$, If $|\overrightarrow{OB}| = 5$ *and* If $|\overrightarrow{OC}| = 6$ then express OP in terms of its component vectors.

(c) If $\vec{a}, \vec{b}$ *and* $\vec{c}$ are the position vectors of vertices A, B and C of ΔABC, then write the position vector of the centroid of ΔABC.

(d) If $\vec{a}$ and $\vec{b}$ are the position vectors of A and B respectively, then find the position vector of the point which divides the join of A and B in the ratio 2 : 3 internally.

Answer – (a) component vectors

(b) $4\hat{\imath} + 5\hat{\jmath} + 6\hat{k}$

(c) $\frac{\vec{a}+\vec{b}+\vec{c}}{3}$

(d) $\frac{3\vec{a}+2\vec{b}}{5}$

Q29 - Read the passage and answer the question that follow it.

Let f be a real function and let c be any point in the domain of f. Then,

(i) c is called the point of local maxima if there exists h > 0 such that f(c) ≥ f(x), for all x ∈ (c − h, c + h). The number f (c) is called the local maximum value of f.

(ii) c is called the point of local minima. If there exists h > 0 such that f(c) ≤ f(x), for all x ∈ (c − h, c + h).

The number f(c) is called the local minimum value of f.

The values of x for which f'(x)=0 are called stationary points or turning points.

The value of x for which f'(x)=0 or f'(x) does not exist are called critical points.

Further, the end of points of domain or f cannot be the points of local maxima or local minima.

Consider the function $f(x) = \left\{\frac{1-x+x^2}{1+x+x^2}\right\}$ on R.

(i) How many critical points does the function f(x) have ?

(a) 0

(b) 1

(c) 2

(d) 3

(ii) Which statement about the critical points of the function f(x) is correct ?

(a) All the critical points of f(x) are positive.

(b) All the critical points of f(x) are negative.

(c) Some critical points of f(x) are positive while others are negative.

(d) None of above as function f(x) does not have any critical point.

(iii) At positive value of critical point of f(x), the function f(x) has :

(a) Local maxima.

(b) Local minima.

(c) Neither local maxima nor local minima.

(d) None of the above as function f(x) have any critical point.

(iv) At negative value of critical point of f(x), the function has :

(a) Local maxima.

(b) Local minima.

(c) Neither local maxima nor local minima

(d) None of the above as function f(x) have any critical point.

(v) Local minimum value of f(x) is :

(a) 3

(b) 1/3

(c) -1

(d) None of these

(vi) Local maximum value of f(x) is :

(a) 7

(b) 3

(c) 5

(d) None of these

Answer -

(i) (c) 2

(ii) (c) Some critical points of f(x) are positive while others are negative.

(iii) (b) Local minima.

(iv) (a) Local maxima.

(v) (b) 1/3

(vi) (b) 3

SECTION - B

Q30 - Find the equation of the hyperbola with vertices at (0, ±6) and $e = \frac{5}{3}$.

Answer –

Step 1: Identify standard form

Vertices at $(0, \pm a)$means transverse axis is along **y-axis**.
So standard equation is:

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

Here $a = 6 \Rightarrow a^2 = 36$.

Step 2: Use eccentricity

For hyperbola:

$$e = \frac{c}{a} \Rightarrow c = ae$$
$$c = 6 \cdot \frac{5}{3} = 10 \Rightarrow c^2 = 100$$

Step 3: Relation among a, b, c

For hyperbola:

$$c^2 = a^2 + b^2$$

So:

$$b^2 = c^2 - a^2 = 100 - 36 = 64$$

Final Answer:

$$\boxed{\frac{y^2}{36} - \frac{x^2}{64} = 1}$$

OR

Find the equation of the ellipse, when focus is (−1, 1), directrix is $x - y + 3 = 0$ and $e = \frac{1}{2}$.

Answer –

Step 1: Use definition of conic

For any point $P(x, y)$on ellipse:

$$PF = e \cdot PD$$

Step 2: Distance to focus $F(-1, 1)$

$$PF = \sqrt{(x+1)^2 + (y-1)^2}$$

Step 3: Distance to directrix $x - y + 3 = 0$

Distance of point $(x, y)$from line $ax + by + c = 0$is:

$$PD = \frac{|ax + by + c|}{\sqrt{a^2 + b^2}}$$

Here $a = 1, b = -1, c = 3$:

$$PD = \frac{|x - y + 3|}{\sqrt{2}}$$

Step 4: Apply $e = \frac{1}{2}$

$$\sqrt{(x+1)^2 + (y-1)^2} = \frac{1}{2} \cdot \frac{|x - y + 3|}{\sqrt{2}}$$

Step 5: Square both sides

$$(x+1)^2 + (y-1)^2 = \frac{1}{8}(x - y + 3)^2$$

Multiply by 8:

$$8[(x+1)^2 + (y-1)^2] = (x - y + 3)^2$$

Expand:

Left:

$$(x+1)^2 = x^2 + 2x + 1, (y-1)^2 = y^2 - 2y + 1$$
$$8(x^2 + y^2 + 2x - 2y + 2) = 8x^2 + 8y^2 + 16x - 16y + 16$$

Right:

$$(x - y + 3)^2 = x^2 + y^2 - 2xy + 6x - 6y + 9$$

Bring all to one side (Left − Right = 0):

$$(8x^2 - x^2) + (8y^2 - y^2) + 2xy + (16x - 6x) + (-16y + 6y) + (16 - 9) = 0$$
$$7x^2 + 7y^2 + 2xy + 10x - 10y + 7 = 0$$

Final Answer:

$$\boxed{7x^2 + 2xy + 7y^2 + 10x - 10y + 7 = 0}$$

Q31 - Solve the matrix equation : $\begin{bmatrix} 2 & -3 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

Answer –

$$\begin{bmatrix} 2 & -3 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

Matrix gives:

$$2x - 3y = 1(1)$$
$$x + y = 3(2)$$

From (2):

$$x = 3 - y$$

Put in (1):

$$2(3 - y) - 3y = 1$$
$$6 - 2y - 3y = 1$$
$$6 - 5y = 1 \Rightarrow 5y = 5 \Rightarrow y = 1$$

Then:

$$x = 3 - 1 = 2$$

Final Answer: $\boxed{x = 2, \quad y = 1}$

OR

Solve for x : $\begin{vmatrix} 0 & 1 & 0 \\ x & 2 & x \\ 1 & 3 & x \end{vmatrix} = 0$

Answer –

$$\begin{vmatrix} 0 & 1 & 0 \\ x & 2 & x \\ 1 & 3 & x \end{vmatrix} = 0$$

Expanding along the first row:

$$= 0\begin{vmatrix} 2 & x \\ 3 & x \end{vmatrix} - 1\begin{vmatrix} x & x \\ 1 & x \end{vmatrix} + 0\begin{vmatrix} x & 2 \\ 1 & 3 \end{vmatrix}$$

$$= -\begin{vmatrix} x & x \\ 1 & x \end{vmatrix}$$

Now evaluate the 2×2 determinant:

$$= -(x \cdot x - x \cdot 1)$$

$$= -(x^2 - x)$$

$$= -x^2 + x$$

Set equal to zero:

$$-x^2 + x = 0$$

$$x(1 - x) = 0$$

$$x = 0 \quad \text{or} \quad x = 1$$

Q32 - Prove that : $\begin{vmatrix} -a^2 & ab & ac \\ ab & -b^2 & bc \\ ac & bc & -c^2 \end{vmatrix} = 4a^2b^2c^2$

Answer –

$$\Delta = \begin{vmatrix} -a^2 & ab & ac \\ ab & -b^2 & bc \\ ac & bc & -c^2 \end{vmatrix}$$

Factor a, b, c from rows:

$$= abc \begin{vmatrix} -a & b & c \\ a & -b & c \\ a & b & -c \end{vmatrix}$$

Now factor a, b, c from columns:

$$= a^2b^2c^2 \begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix}$$

Now evaluate the determinant:

$$\begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix}$$

Expand along first row:

$$= -1 \begin{vmatrix} -1 & 1 \\ 1 & -1 \end{vmatrix} - 1 \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} + 1 \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix}$$

Compute minors:

$$\begin{vmatrix} -1 & 1 \\ 1 & -1 \end{vmatrix} = (-1)(-1) - (1)(1) = 1 - 1 = 0$$

$$\begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} = (1)(-1) - (1)(1) = -1 - 1 = -2$$

$$\begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} = (1)(1) - (-1)(1) = 1 + 1 = 2$$

Substitute:

$$= -1(0) - 1(-2) + 1(2)$$

$$= 0 + 2 + 2$$

$$= 4$$

Therefore,

$$\Delta = a^2b^2c^2 \cdot 4$$

$$\boxed{\Delta = 4a^2b^2c^2}$$

Q33 – Simplify : $tan[sin^{-1}(\sqrt{1-x})]$

Answer –

$$\tan(\sin^{-1}(\sqrt{1-x}))$$

Let:

$$\theta = \sin^{-1}(\sqrt{1-x}) \Rightarrow \sin\theta = \sqrt{1-x}$$

Then:

$$\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-(1-x)} = \sqrt{x}$$

So:

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\sqrt{1-x}}{\sqrt{x}} = \sqrt{\frac{1-x}{x}}$$

Final Answer:

$$\boxed{\sqrt{\frac{1-x}{x}}}$$

Q34 – Evaluate : $\lim_{x\to3}\frac{|x-3|}{x-3}$

Answer –

$$\lim_{x\to3}\frac{|x-3|}{x-3}$$

If $x > 3$: $|x-3| = x-3 \Rightarrow \frac{|x-3|}{x-3} = 1$

If $x < 3$: $|x-3| = -(x-3) \Rightarrow \frac{|x-3|}{x-3} = -1$

Left limit = -1, right limit = $1 \Rightarrow$ not equal $\Rightarrow$ limit does not exist.

Final Answer: $\boxed{\text{Does not exist}}$

OR

Evaluate : $\lim_{x\to 0}\frac{\sin 3x}{x}$

Answer –

$$\lim_{x\to 0}\frac{\sin 3x}{x}$$

Rewrite the expression:

$$= \lim_{x\to 0}\left(\frac{\sin 3x}{3x}\cdot\frac{3x}{x}\right)$$

$$= \lim_{x\to 0}\left(\frac{\sin 3x}{3x}\cdot 3\right)$$

Using the standard limit:

$$\lim_{t\to 0}\frac{\sin t}{t} = 1$$

Let $t = 3x$. As $x \to 0$, $t \to 0$.

$$\lim_{x\to 0}\frac{\sin 3x}{3x} = 1$$

Therefore,

$$= 1\cdot 3$$

$$= 3$$

Q35 – If $y = \frac{x+1}{x-1}$, $(x \neq 1)$ find $\frac{d^2y}{dx^2}$.

Answer –

Rewrite:

$$\frac{x+1}{x-1} = 1 + \frac{2}{x-1}$$

First derivative:

$$\frac{dy}{dx} = 0 + 2\frac{d}{dx}(x-1)^{-1} = 2(-1)(x-1)^{-2} = -\frac{2}{(x-1)^2}$$

Second derivative:

$$\frac{d^2y}{dx^2} = \frac{d}{dx}(-2(x-1)^{-2}) = -2(-2)(x-1)^{-3} = \frac{4}{(x-1)^3}$$

Final Answer: $\boxed{\frac{4}{(x-1)^3}}$

OR

If $y = \frac{1}{x}e^x$, Find $\frac{dy}{dx}$.

Answer –

$$y = e^x \cdot x^{-1}$$

Product rule:

$$\frac{dy}{dx} = e^x \cdot x^{-1} + e^x \cdot (-1)x^{-2} = e^x\left(\frac{1}{x} - \frac{1}{x^2}\right) = \frac{e^x(x-1)}{x^2}$$

Final Answer: $\boxed{\frac{e^x(x-1)}{x^2}}$

Q36 – If $cosy = xcos(a+y)$, then prove that $\frac{dy}{dx} = \frac{cos^2(a+y)}{sin\ a}$.

Answer –

Given:

$$\cos y = x\cos(a+y)$$

Prove:

$$\frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$$

Differentiate both sides:

Left:

$$\frac{d}{dx}(\cos y) = -\sin y \cdot \frac{dy}{dx}$$

Right (product rule):

$$\frac{d}{dx}[x\cos(a+y)] = \cos(a+y) + x[-\sin(a+y)]\frac{dy}{dx}$$

So:

$$-\sin y\, y' = \cos(a+y) - x\sin(a+y)\, y'$$

Bring y' terms together:

$$y'[-\sin y + x\sin(a+y)] = \cos(a+y)$$

From original equation:

$$x = \frac{\cos y}{\cos(a+y)}$$

Substitute:

$$-\sin y + \frac{\cos y}{\cos(a+y)}\sin(a+y) = \frac{-\sin y\cos(a+y) + \cos y\sin(a+y)}{\cos(a+y)}$$

Numerator:

$$\sin(a+y-y) = \sin a$$

So bracket becomes:

$$\frac{\sin a}{\cos(a+y)}$$

Thus:

$$y' \cdot \frac{\sin a}{\cos(a+y)} = \cos(a+y) \Rightarrow y' = \frac{\cos^2(a+y)}{\sin a}$$

Proved.

Q37 - Find the distance of the point (3, 4, −5) from the plane $2x - 3y + 3z + 27 = 0$.

Answer -

Distance of point $(3,4,-5)$from plane $2x - 3y + 3z + 27 = 0$

Formula:

$$D = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

Here $a = 2, b = -3, c = 3, d = 27$

Numerator:

$$|2(3) + (-3)(4) + 3(-5) + 27| = |6 - 12 - 15 + 27| = |6| = 6$$

Denominator:

$$\sqrt{4 + 9 + 9} = \sqrt{22}$$

Final Answer: $\boxed{\frac{6}{\sqrt{22}}}$

Q38 - Reduce the equation of the plane $4x - 5y + 6z - 60 = 0$ to the intercept form. Find its intercepts on the coordinate axes.

Answer –

Reduce plane $4x - 5y + 6z - 60 = 0$to intercept form and find intercepts.

$$4x - 5y + 6z = 60$$

Divide by 60:

$$\frac{x}{15} - \frac{y}{12} + \frac{z}{10} = 1$$

Intercepts:

- x-intercept: $x = 15$
- y-intercept: $y = -12$
- z-intercept: $z = 10$

Final Answer:

$$\boxed{\frac{x}{15} - \frac{y}{12} + \frac{z}{10} = 1; (15,0,0), (0,-12,0), (0,0,10)}$$

Q39 - For the ellipse $3x^2 + 2y^2 = 6$, find the lengths of minor and major axes, coordinate of foci, vertices and the eccentricity.

Answer -

Step 1: Convert to standard ellipse form

Given:

$$3x^2 + 2y^2 = 6$$

Divide both sides by 6:

$$\frac{3x^2}{6} + \frac{2y^2}{6} = 1$$
$$\frac{x^2}{2} + \frac{y^2}{3} = 1$$

Now compare with standard ellipse:

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

where $a^2$is the **bigger denominator** (major axis direction).

Here:

$$a^2 = 3, b^2 = 2$$

So:

$$a = \sqrt{3}, b = \sqrt{2}$$

☑ Since $a^2$is under y^2, the **major axis is along y-axis**.

Step 2: Lengths of axes

Formulas:

- Major axis length $= 2a$
- Minor axis length $= 2b$

So:

$$\text{Major axis length} = 2\sqrt{3}$$
$$\text{Minor axis length} = 2\sqrt{2}$$

Step 3: Find c for foci

Formula for ellipse:

$$c^2 = a^2 - b^2$$
$$c^2 = 3 - 2 = 1 \Rightarrow c = 1$$

Step 4: Coordinates of foci

Since major axis is along y-axis, foci are:

$$(0, \pm c) = (0, \pm 1)$$

Foci: $\boxed{(0,1) \text{ and } (0,-1)}$

Step 5: Vertices

Vertices on major axis (y-axis):

$$(0, \pm a) = (0, \pm\sqrt{3})$$

Co-vertices on minor axis (x-axis):

$$(\pm b, 0) = (\pm\sqrt{2}, 0)$$

Vertices (major): $\boxed{(0,\sqrt{3}), (0,-\sqrt{3})}$

Co-vertices (minor): $\boxed{(\sqrt{2},0), (-\sqrt{2},0)}$

Step 6: Eccentricity

Formula:

$$e = \frac{c}{a}$$
$$e = \frac{1}{\sqrt{3}}$$

(If you want rationalized form:)

$$e = \frac{\sqrt{3}}{3}$$

Eccentricity: $\boxed{\frac{1}{\sqrt{3}}}$(or $\boxed{\frac{\sqrt{3}}{3}}$)

Q40 - Using elementary column transformation, fluid the inverse of the matrix $A = \begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix}$.

Answer –

Step 1: Determinant

- For 2×2, $|A| = ad - bc$

Here $a = 2, b =- 6, c = 1, d =- 2$

$$|A| = 2(-2) - (-6)(1) =- 4 + 6 = 2$$

Since determinant $\neq 0$, inverse exists ☑

Step 2: Adjoint matrix

- For $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$,

$$\text{adj}(A) = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

So:

$$\text{adj}(A) = \begin{bmatrix} -2 & 6 \\ -1 & 2 \end{bmatrix}$$

Step 3: Inverse formula

-

$$A^{-1} = \frac{1}{|A|}\text{adj}(A)$$
$$A^{-1} = \frac{1}{2}\begin{bmatrix} -2 & 6 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} -1 & 3 \\ -\frac{1}{2} & 1 \end{bmatrix}$$

$$\boxed{A^{-1} = \begin{bmatrix} -1 & 3 \\ -\frac{1}{2} & 1 \end{bmatrix}}$$

Q41 - Prove that : $tan^{-1}\left(\frac{1}{7}\right) + tan^{-1}\left(\frac{1}{13}\right) = tan^{-1}\left(\frac{2}{9}\right)$.

Answer –

$$\tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left(\frac{1}{13}\right) = \tan^{-1}\left(\frac{2}{9}\right)$$

Step 1: Use the addition formula for inverse tan

Formula:

$$\tan^{-1}a + \tan^{-1}b = \tan^{-1}\left(\frac{a+b}{1-ab}\right)$$

(when $ab < 1$and angles lie in principal range — here it's fine because numbers are small and positive)

Let:

$$a = \frac{1}{7}, b = \frac{1}{13}$$

Step 2: Compute numerator $a + b$

$$a + b = \frac{1}{7} + \frac{1}{13} = \frac{13+7}{91} = \frac{20}{91}$$

Step 3: Compute denominator $1 - ab$

$$ab = \frac{1}{7} \cdot \frac{1}{13} = \frac{1}{91}$$

Step 4: Put into formula

$$\tan^{-1}\left(\frac{a+b}{1-ab}\right) = \tan^{-1}\left(\frac{\frac{20}{91}}{\frac{90}{91}}\right)$$
$$= \tan^{-1}\left(\frac{20}{90}\right) = \tan^{-1}\left(\frac{2}{9}\right)$$

So,

$$\tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left(\frac{1}{13}\right) = \tan^{-1}\left(\frac{2}{9}\right)$$

Proved ☑

Q42 - Find the value of $\int_0^{\pi/2} \frac{sinx-cosx}{1+sinxcosx} dx$.

Answer –

Let

$$I = \int_0^{\pi/2} f(x)\, dx$$

Now substitute $x \to \frac{\pi}{2} - x$.

Since the limits remain 0 to $\pi/2$,

$$I = \int_0^{\pi/2} \frac{\sin(\frac{\pi}{2} - x) - \cos(\frac{\pi}{2} - x)}{1 + \sin(\frac{\pi}{2} - x)\cos(\frac{\pi}{2} - x)}\, dx$$

Using identities:

$$\sin(\tfrac{\pi}{2} - x) = \cos x, \quad \cos(\tfrac{\pi}{2} - x) = \sin x$$

$$I = \int_0^{\pi/2} \frac{\cos x - \sin x}{1 + \sin x \cos x}\, dx$$

$$I = -\int_0^{\pi/2} \frac{\sin x - \cos x}{1 + \sin x \cos x}\, dx$$

$$I = -I$$

Therefore,

$$2I = 0$$

$$\boxed{I = 0}$$

OR

Evaluate : $\int_{-3}^{3} |x + 1| dx$.

Answer –

The expression $x + 1 = 0$ at $x = -1$.

So split the integral at $x = -1$.

For $x < -1$, $x + 1 < 0 \Rightarrow |x + 1| = -(x + 1)$.

For $x > -1$, $x + 1 > 0 \Rightarrow |x + 1| = x + 1$.

$$I = \int_{-3}^{-1} -(x + 1)\, dx + \int_{-1}^{3} (x + 1)\, dx$$

First integral:

$$\int_{-3}^{-1} -(x + 1)\, dx = \int_{-3}^{-1} (-x - 1)\, dx$$

$$= \left[-\frac{x^2}{2} - x \right]_{-3}^{-1}$$

At $x = -1$:

$$-\frac{1}{2} + 1 = \frac{1}{2}$$

At $x = -3$:

$$-\frac{9}{2} + 3 = -\frac{3}{2}$$

$$= \frac{1}{2} - \left(-\frac{3}{2}\right) = 2$$

Second integral:

$$\int_{-1}^{3} (x+1)\,dx = \left[\frac{x^2}{2} + x\right]_{-1}^{3}$$

At $x = 3$:

$$\frac{9}{2} + 3 = \frac{15}{2}$$

At $x = -1$:

$$\frac{1}{2} - 1 = -\frac{1}{2}$$

$$= \frac{15}{2} - \left(-\frac{1}{2}\right) = 8$$

Therefore,

$$\boxed{I = 2 + 8 = 10}$$

Q43 - Prove that the lines

$\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$ and $\frac{x-2}{1} = \frac{y-4}{4} = \frac{z-6}{7}$ are coplanar. Also, find the equation of the plane containing these lines.

Answer –

Step 1: Write in parametric form

For first line L_1, let parameter t:

$$x = -1 + 3t, \quad y = -3 + 5t, \quad z = -5 + 7t$$

Point on L_1:

$$A(-1, -3, -5)$$

Direction vector:

$$\vec{d}_1 = \langle 3, 5, 7 \rangle$$

For second line L_2, let parameter s:

$$x = 2 + s, \quad y = 4 + 4s, \quad z = 6 + 7s$$

Point on L_2:

$$B(2, 4, 6)$$

Direction vector:

$$\vec{d}_2 = \langle 1, 4, 7 \rangle$$

Step 2: Check coplanarity

Vector joining points A and B:

$$\vec{AB} = B - A = (3, 7, 11)$$

Now compute scalar triple product:

$$\vec{AB} \cdot (\vec{d}_1 \times \vec{d}_2)$$

First find cross product:

$$\vec{d}_1 \times \vec{d}_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 5 & 7 \\ 1 & 4 & 7 \end{vmatrix}$$

$$= \mathbf{i}(35 - 28) - \mathbf{j}(21 - 7) + \mathbf{k}(12 - 5)$$

$$= (7, -14, 7)$$

Now dot with $\vec{AB}$:

$$(3, 7, 11) \cdot (7, -14, 7)$$

$$= 21 - 98 + 77$$

$$= 0$$

Since scalar triple product = 0,

the lines are **coplanar**.

Step 3: Equation of the plane

Normal vector of plane:

$$\vec{n} = \vec{d_1} \times \vec{d_2} = (7, -14, 7)$$

Divide by 7:

$$\vec{n} = (1, -2, 1)$$

Using point $A(-1, -3, -5)$:

$$1(x+1) - 2(y+3) + 1(z+5) = 0$$

$$x + 1 - 2y - 6 + z + 5 = 0$$

$$x - 2y + z = 0$$

Final Answer

The lines are **coplanar** and the equation of the plane is

$$\boxed{x - 2y + z = 0}$$

Q44 - A machine producing either products A or B can produce one unit of A by using 2 units of chemicals and 1 unit of a compound and can produce one unit of B by using 1 unit of chemicals and 2 units of the compound. Only 800 units of chemicals and 1000 units of the compound are available. The profit available per unit of A and B are respectively ₹30 and ₹20. Find the optimum allocation of units between A and B to maximise, the total profit. Find the maximum profit.

Answer –

Let

x = number of units of product A
y = number of units of product B

Step 1: Form constraints

Chemicals constraint:

$$2x + y \leq 800$$

Compound constraint:

$$x + 2y \leq 1000$$

Also,

$$x \geq 0, \quad y \geq 0$$

Step 2: Objective function

Profit:

$$Z = 30x + 20y$$

We need to maximize Z.

Step 3: Find corner points of feasible region

From

$$2x + y = 800$$

$$x + 2y = 1000$$

Solve simultaneously.

From first:

$$y = 800 - 2x$$

Substitute into second:

$$x + 2(800 - 2x) = 1000$$

$$x + 1600 - 4x = 1000$$

$$-3x = -600$$

$$x = 200$$

$$y = 800 - 400 = 400$$

Intersection point:

$$(200, 400)$$

Other corner points:

When $x = 0$:

$$x + 2y = 1000 \Rightarrow y = 500$$

Point: $(0, 500)$

When $y = 0$:

$$2x = 800 \Rightarrow x = 400$$

Point: $(400, 0)$

Also $(0, 0)$.

Step 4: Evaluate profit at corner points

$$Z(0,0) = 0$$

$$Z(400,0) = 30(400) = 12000$$

$$Z(0,500) = 20(500) = 10000$$

$$Z(200,400) = 30(200) + 20(400)$$

$$= 6000 + 8000$$

$$= 14000$$

Final Answer

Optimum production:

$$\boxed{x = 200, \quad y = 400}$$

Maximum profit:

$$\boxed{₹14,000}$$

OR

A manufacturer produces nuts and bolts. It takes 1 hour of work on machine A and 3 hours on machine B to produce a package of nuts. It takes 3 hours on machine A and 1 hour on machine B to produce a package of bolts. He earns a profit of ₹17.50 per package on nuts and ₹7 per package on botts. How many packages of each should be produced each day so as to maximise his profits if he operates his machine for at the most 12 hours a day? Form the above as a L.P.P. and then solve it graphically

Answer -

Let

x = number of packages of nuts per day
y = number of packages of bolts per day

Formulation of the L.P.P.

Time on machine A:

$$1x + 3y \leq 12$$

Time on machine B:

$$3x + 1y \leq 12$$

Non-negativity restrictions:

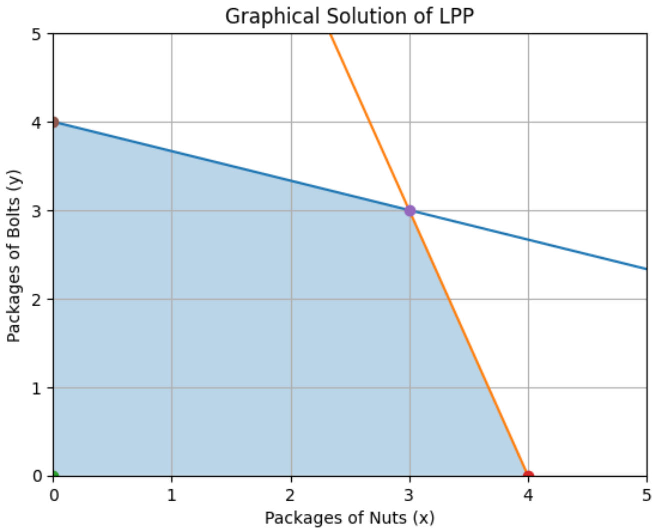
$$x \geq 0, \quad y \geq 0$$

Objective function (profit):

$$Z = 17.5x + 7y$$

Maximise Z.

Graphical Solution (Corner Point Method)



Feasible region is determined by the system:

$$x + 3y \leq 12$$

$$3x + y \leq 12$$

$$x \geq 0, \quad y \geq 0$$

Corner points of the feasible region:

$$(0, 0), \quad (4, 0), \quad (3, 3), \quad (0, 4)$$

Objective function:

$$Z = 17.5x + 7y$$

Evaluating at corner points:

$$Z(0,0) = 17.5(0) + 7(0) = 0$$

$$Z(4,0) = 17.5(4) + 7(0) = 70$$

$$Z(0,4) = 17.5(0) + 7(4) = 28$$

$$Z(3,3) = 17.5(3) + 7(3) = 52.5 + 21 = 73.5$$

Since

$$\max Z = 73.5 \text{ at } (x, y) = (3, 3),$$

the optimal solution is

$$x = 3, \quad y = 3$$

Maximum profit:

$$Z_{\max} = ₹\,73.50$$

Q45 - Show that of all the rectangles of given perimeter, the square has the greatest area.

Answer -

Let the given perimeter be P.

Let the length be l and breadth be b.

Then,

$$2(l + b) = P$$

$$l + b = \frac{P}{2}$$

Area of rectangle:

$$A = lb$$

Express b in terms of l:

$$b = \frac{P}{2} - l$$

Substitute in area:

$$A = l\left(\frac{P}{2} - l\right)$$

$$A = \frac{P}{2}l - l^2$$

$$A = -l^2 + \frac{P}{2}l$$

This is a quadratic in l opening downward, so maximum area occurs at the vertex.

Differentiate:

$$\frac{dA}{dl} = -2l + \frac{P}{2}$$

Set equal to zero:

$$-2l + \frac{P}{2} = 0$$

$$2l = \frac{P}{2}$$

$$l = \frac{P}{4}$$

Then,

$$b = \frac{P}{2} - \frac{P}{4}$$

$$b = \frac{P}{4}$$

Thus,

$$l = b$$

Hence, the rectangle becomes a square.

Maximum area:

$$A_{\max} = \left(\frac{P}{4}\right)^2 = \frac{P^2}{16}$$

Therefore, of all rectangles with a given perimeter, the square has the greatest area.

OR

Find the point on the curve $y^2 = 4x$ which is nearest to the point (z, 1).

Answer –

Given curve:

$$y^2 = 4x$$

Let the point on the curve be $P(x, y)$.

Since $y^2 = 4x$,

$$x = \frac{y^2}{4}$$

Given external point is $(2, 1)$.

Distance squared from $P(x, y)$ to $(2, 1)$:

$$D^2 = (x - 2)^2 + (y - 1)^2$$

Substitute $x = \frac{y^2}{4}$:

$$D^2 = \left(\frac{y^2}{4} - 2\right)^2 + (y - 1)^2$$

Let

$$F(y) = \left(\frac{y^2}{4} - 2\right)^2 + (y - 1)^2$$

Differentiate:

$$\frac{dF}{dy} = 2\left(\frac{y^2}{4} - 2\right)\left(\frac{y}{2}\right) + 2(y - 1)$$

$$= y\left(\frac{y^2}{4} - 2\right) + 2(y - 1)$$

Set equal to zero:

$$y\left(\frac{y^2}{4} - 2\right) + 2(y - 1) = 0$$

Multiply by 4:

$$y(y^2 - 8) + 8(y - 1) = 0$$

$$y^3 - 8y + 8y - 8 = 0$$

$$y^3 - 8 = 0$$

$$y = 2$$

Now,

$$x = \frac{y^2}{4} = \frac{4}{4} = 1$$

Therefore, the nearest point on the curve is

$$\boxed{(1, 2)}$$