Generic Elective (GE)

Mathematics : Theory of Equations and Symmetries

Total Marks: 100 (Theory: 75, Internal Assessment: 25) **Examination:** 3 Hrs.
Credits: 4

Course Objectives: The goal of this paper is to acquaint students with certain ideas about integral roots, rational roots, an upper bound on number of positive or negative roots of a polynomial, and finding roots of cubic and quartic equations in special cases using elementary symmetric functions and in general using Cardon's and Descartes' methods, respectively.

Course Learning Outcomes: After completion of this paper, the students will be able to:

i) Understand the nature of the roots of polynomial equations and their symmetries.
ii) Solve cubic and quartic polynomial equations with special condition on roots and in general.
iii) Find symmetric functions in terms of the elementary symmetric polynomials.

Unit 1: Polynomial Equations and Properties
General properties of polynomials and equations; Fundamental theorem of algebra and its consequences; Theorems on imaginary, integral and rational roots; Descartes' rule of signs for positive and negative roots; Relations between the roots and coefficients of equations, Applications to solution of equations when an additional relation among the roots is given; De Moivre's theorem for rational indices, the *n*th roots of unity and symmetries of the solutions.

Unit 2: Cubic and Biquadratic (Quartic) Equations
Transformation of equations (multiplication, reciprocal, increase/diminish in the roots by a given quantity), Removal of terms; Cardon's method of solving cubic and Descartes' method of solving biquadratic equations.

Unit 3: Symmetric Functions
Elementary symmetric functions and symmetric functions of the roots of an equation; Newton's theorem on sums of the like powers of the roots; Computation of symmetric functions such as $\sum\alpha^2\beta,\ \sum\alpha^2\beta^2, \sum\alpha^2\beta\gamma, \sum\frac{1}{\alpha^2\beta\gamma}, \sum\alpha^{-3}, \sum(\beta+\gamma-\alpha)^2,\ \sum\frac{\alpha^2+\beta\gamma}{\beta+\gamma},\ldots$ of polynomial equations; Transformation of equations by symmetric functions and in general.

References:

1. Burnside, W.S., & Panton, A.W. (1979). *The Theory of Equations* (11th ed.). Vol. 1. Dover Publications, Inc. (4th Indian reprint. S. Chand & Co. New Delhi).
2. Dickson, Leonard Eugene (2009). *First Course in the Theory of Equations*. John Wiley & Sons, Inc. The Project Gutenberg eBook: http://www.gutenberg.org/ebooks/29785

Additional Reading:

i. Prasad, Chandrika (2017). *Text Book of Algebra and Theory of Equations*. Pothishala Pvt Ltd.