

Generic Elective (GE)

Mathematics : Theory of Equations and Symmetries

Total Marks: 100 (Theory: 75, Internal Assessment: 25) **Examination: 3 Hrs.**
Credits: 4

Course Objectives: The goal of this paper is to acquaint students with certain ideas about integral roots, rational roots, an upper bound on number of positive or negative roots of a polynomial, and finding roots of cubic and quartic equations in special cases using elementary symmetric functions and in general using Cardon's and Descartes' methods, respectively.

Course Learning Outcomes: After completion of this paper, the students will be able to:

- Understand the nature of the roots of polynomial equations and their symmetries.
- Solve cubic and quartic polynomial equations with special condition on roots and in general.
- Find symmetric functions in terms of the elementary symmetric polynomials.

Unit 1: Polynomial Equations and Properties

General properties of polynomials and equations; Fundamental theorem of algebra and its consequences; Theorems on imaginary, integral and rational roots; Descartes' rule of signs for positive and negative roots; Relations between the roots and coefficients of equations, Applications to solution of equations when an additional relation among the roots is given; De Moivre's theorem for rational indices, the n th roots of unity and symmetries of the solutions.

Unit 2: Cubic and Biquadratic (Quartic) Equations

Transformation of equations (multiplication, reciprocal, increase/diminish in the roots by a given quantity), Removal of terms; Cardon's method of solving cubic and Descartes' method of solving biquadratic equations.

Unit 3: Symmetric Functions

Elementary symmetric functions and symmetric functions of the roots of an equation; Newton's theorem on sums of the like powers of the roots; Computation of symmetric functions such as

$\sum \alpha^2 \beta$, $\sum \alpha^2 \beta^2$, $\sum \alpha^2 \beta \gamma$, $\sum \frac{1}{\alpha^2 \beta \gamma}$, $\sum \alpha^{-3}$, $\sum (\beta + \gamma - \alpha)^2$, $\sum \frac{\alpha^2 + \beta \gamma}{\beta + \gamma}$, ... of polynomial equations;

Transformation of equations by symmetric functions and in general.

References:

- Burnside, W.S., & Panton, A.W. (1979). *The Theory of Equations* (11th ed.). Vol. 1. Dover Publications, Inc. (4th Indian reprint. S. Chand & Co. New Delhi).
- Dickson, Leonard Eugene (2009). *First Course in the Theory of Equations*. John Wiley & Sons, Inc. The Project Gutenberg eBook: <http://www.gutenberg.org/ebooks/29785>

Additional Reading:

- Prasad, Chandrika (2017). *Text Book of Algebra and Theory of Equations*. Pothishala Pvt Ltd.